Fuzzy rules

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What is an inference rule?

We can consider a rule as a **model**, a way to define a **mapping** from input to output.

We consider forward chaining rules, having the shape

IF <antecedent> THEN <consequent>

where:

- <antecedent> is a set of <clause>s related by logical operators AND, OR, NOT
- <consequent> is a set of <clause>s often related by the only logical operator AND
- <clause> in general is either a <proposition>, a sequence of symbols, or a <pattern>, i.e. a sequence of symbols and variables

What are rules for?

Rules are used to represent inferential relationships among pieces of knowledge.

They are used to implement Knowledge-Based Systems, among which Expert Systems are mostly known as successful AI applications, e.g. for diagnosis, forecast, design, control, etc.

What is a fuzzy rule?

A **fuzzy rule** is a rule whose clauses have the shape

(V is L)

where V is a linguistic variable and L is a label, a value for V associated to a fuzzy set. This is a **linguistic clause**.

Usually, clauses in the antecedent are only related by the AND operator.

The antecedent is usually **matched** against facts that are represented as values of real-valued variables corresponding to the linguistic variables

The consequent may be one of two types ...

Linguistic rules

Linguistic rules (<u>Mamdani</u>): the consequent is a conjunction of linguistic clauses

IF (A is LA_n) AND (B is LB_k) AND... THEN (U is LU_m) AND ...

E.g.:

IF (Distance is Far) AND (BallDirection is Front)

THEN (Speed id High) AND (Direction is Ahead)

This can be considered as a mapping between

the **interpretation of an input configuration**

and

a **symbolic description of the desired output**

Model rules

Model rules (Sugeno, or Takagi-Sugeno-Kosko TSK):

bind a **model** (linear, non linear, NN, ...) to the linguistic interpretation of its **applicability conditions**

IF (A is LA$_n$) AND (B is LB$_k$) AND... THEN U is f(A, B)

E.g.:

IF (Temperature is High) AND (Pressure is High)
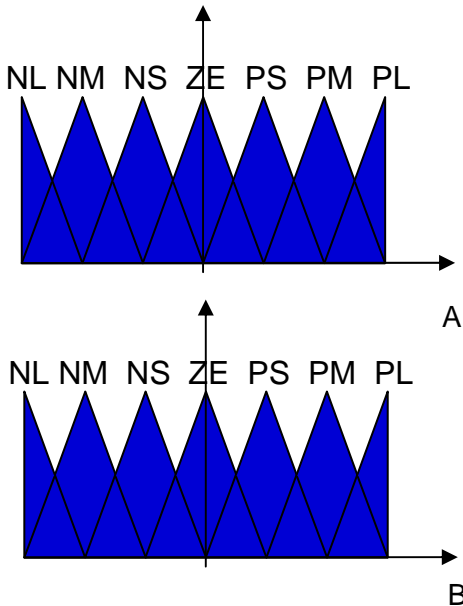
THEN Heating = 2000 - 3T -7P

This can be considered as a mapping between

the **interpretation of an input configuration (the applicability condition of a model)**

and

a **model to be applied to the input real values to obtain the output**

How to use fuzzy rules (Mamdani)

NL NM NS ZE PS PM PL

A

NL NM NS ZE PS PM PL

B

R1: if A is PL and B is PS Then X is PM

R2: if A is PM and B is PS Then X is PS

R3: if A is PL and B is PM Then X is PM
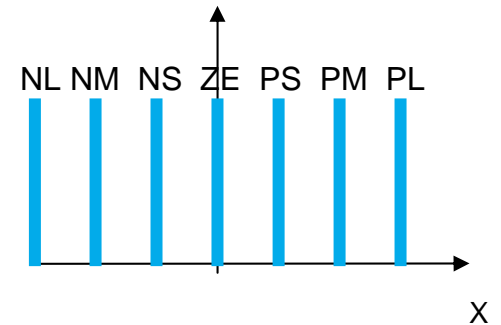
NL NM NS ZE PS PM PL
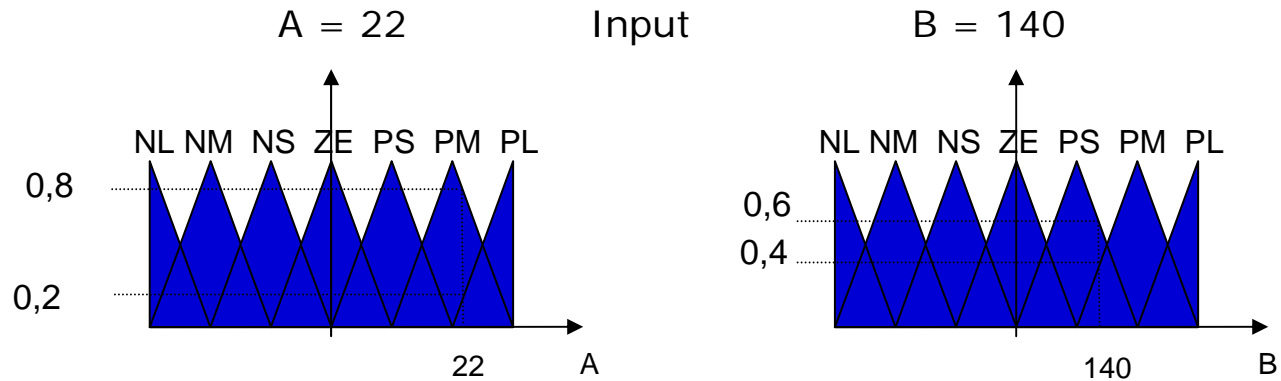
X

Input matching

Combination of matching degrees

Eventual combination with rule weight

Aggregation of output from different rules
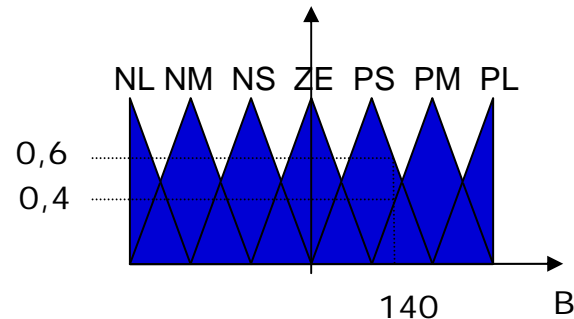
Eventual defuzzyfication of output

Input matching

A = 22   Input   B = 140

NL NM NS ZE PS PM PL

0,8

0,2

22   A

NL NM NS ZE PS PM PL

0,6

0,4

140   B

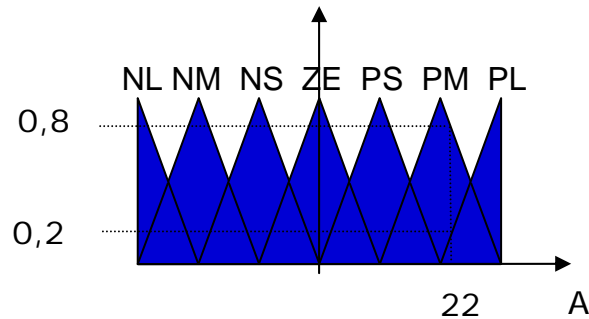R1: IF (A is PL) (B is PS) THEN (X is PM)
         0,2   0,6

R2: IF (A is PM) (B is PS) THEN (X is PS)
         0,8   0,6

R3: IF (A is PM) (B is PM) THEN (X is PM)
         0,8   0,4

Combination of matching degrees in the antecedent

NL NM NS ZE PS PM PL

0,8

0,2

22   A

NL NM NS ZE PS PM PL

0,6
0,4

140   B

We use the **min** operator

R1: IF (A is PL) (B is PS) THEN (X is PM)

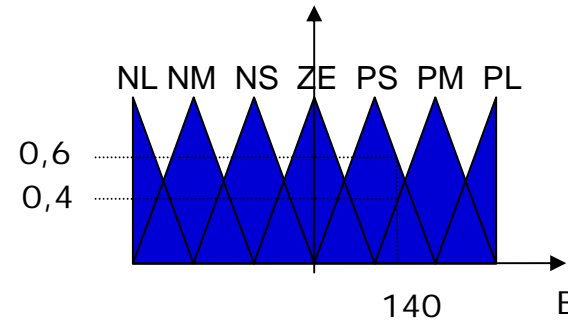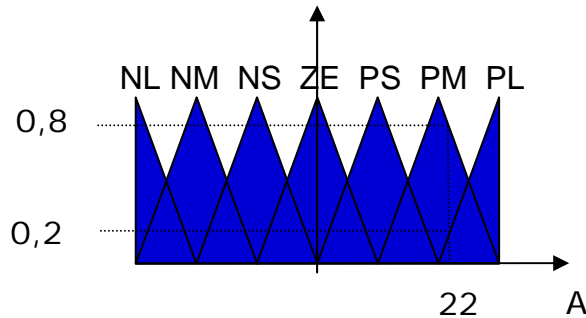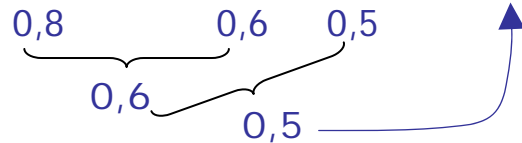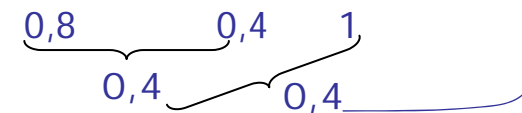 0,2   0,6

 0,2

R2: IF (A is PM) (B is PS) THEN (X is PS)

 0,8   0,6

 0,6

The matching degree of the antecedent is a measure of how the rule fits the current situation

R3: IF (A is PM) (B is PM) THEN (X is PM)

 0,8   0,4

 0,4

Combination with the rule weight

NL NM NS ZE PS PM PL

0,8

0,2

22   A

NL NM NS ZE PS PM PL

0,6

0,4

140   B

We use the **min** operator

R1: IF (A is PL) (B is PS) THEN (X is PM)

0,2   0,6   1

0,2

0,2

R2: IF (A is PM) (B is PS) THEN (X is PS)

0,8   0,6   0,5

0,6

0,5

R3: IF (A is PM) (B is PM) THEN (X is PM)

0,8   0,4   1

0,4

0,4

The resulting value is a measure of how much the rule is "good" given the situation and "per se"

Output aggregation

R1: if (A is PL) (B is PS) Then (X is PM)
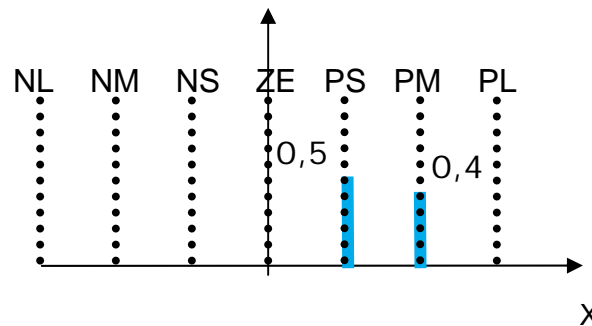
0,2

R2: if (A is PM) (B is PS) Then (X is PS)

0,5

R3: if (A is PL) (B is PM) Then (X is PM)

0,4

X is PM with weight 0,4 ← **max**

X is PS with weight 0,5

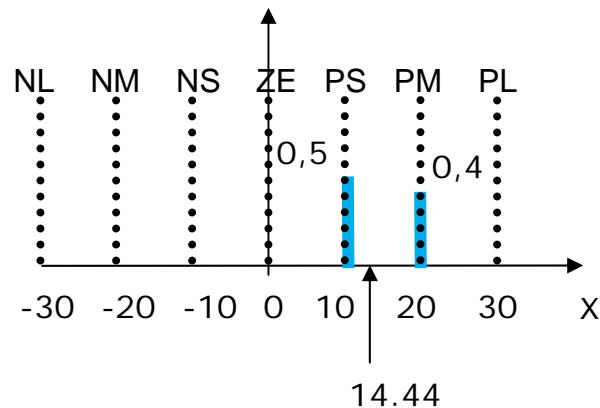We use the operator **max** to aggregate weights given to the same output value

NL   NM   NS   ZE   PS   PM   PL

0,5   0,4

X

(Eventual) defuzzyfication

X is PM with weight 0,4

X is PS with weight 0,5

We use the operator "weighted media" on the weights of the output values to obtain a numerical values

(10*0,5 + 20*0,4)/(0,5 + 0,4) = **14.44**

Defuzzyfication

Also for the defuzzyfication we have many possible operators
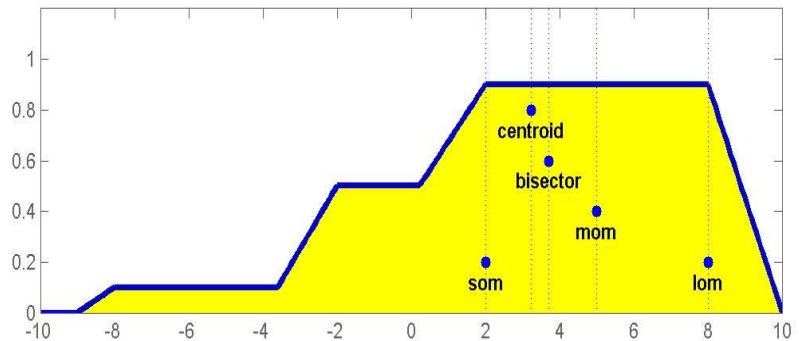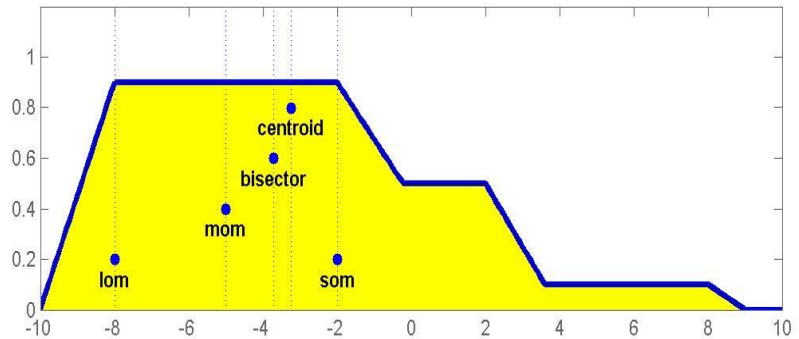
Centroid

Bisector

Average of maxima

Lowest maximum

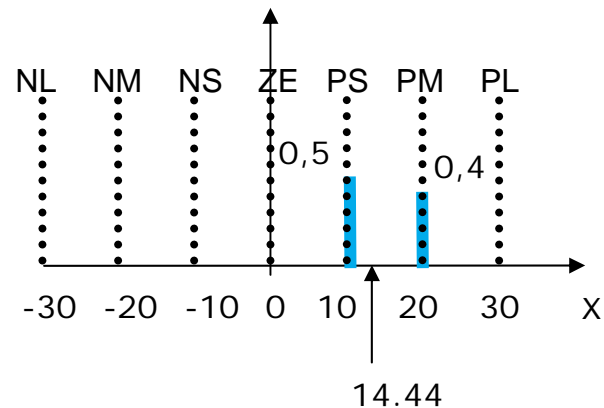Highest maximum

Center of the highest area

…

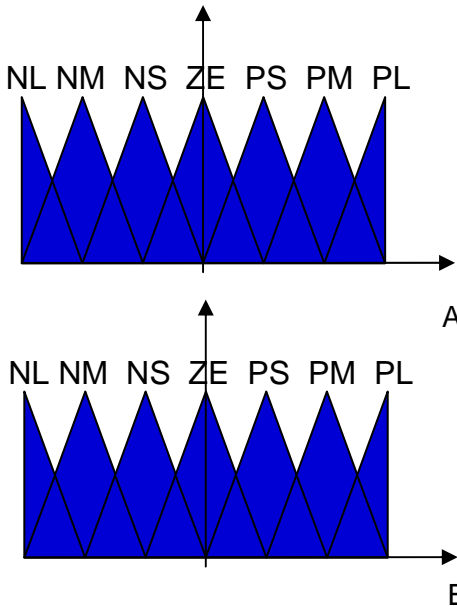# Another possible output: linguistic approximation

X is PM with weight 0,4

X is PS with weight 0,5

$(10*0,5 + 20*0,4)/(0,5 + 0,4) =$ **14.44**

…so **X is PS**

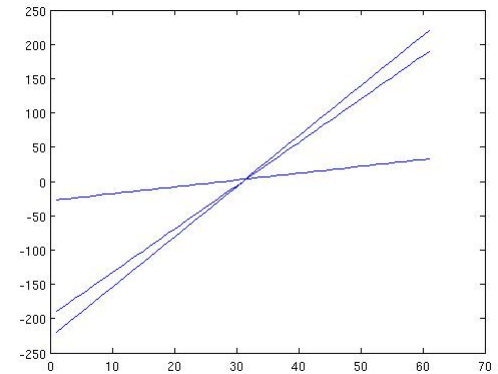# How to use fuzzy rules (Sugeno)

NL NM NS ZE PS PM PL

A

R1: if A is PL and B is PS Then X is A+2B

R2: if A is PM and B is PS Then X is A +3

NL NM NS ZE PS PM PL

R3: if A is PL and B is PM Then X is A+B

B

Input matching

Combination of matching degrees

Eventual combination with rule weight

Aggregation of output from different rules

Eventual defuzzyfication of output

Same as

with

Mamdani

Output aggregation

R1: if (A is PL) (B is PS) Then (X is A+2B)

0.2

R2: if (A is PM) (B is PS) Then (X is A+3)

0.5

R3: if (A is PL) (B is PM) Then (X is A+B)

0.4

X is (0.2*(A+2B)+0.5*(A+3)+0.4*(A+B))/(0.2+0.5+0.4)

Since A=22 and B=140 then X=125.18

This is a weighted combination of the models expressed in the output of the rules matching the inputs

Some exercises

Define the rules to control the light in your room according to the external light coming from a window

Define the rules to decide what to do when you see a large, bad-looking dog on your way