POLITECNICO
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Artificial Neural Networks and Deep Learning

- Recurrent Neural Networks-

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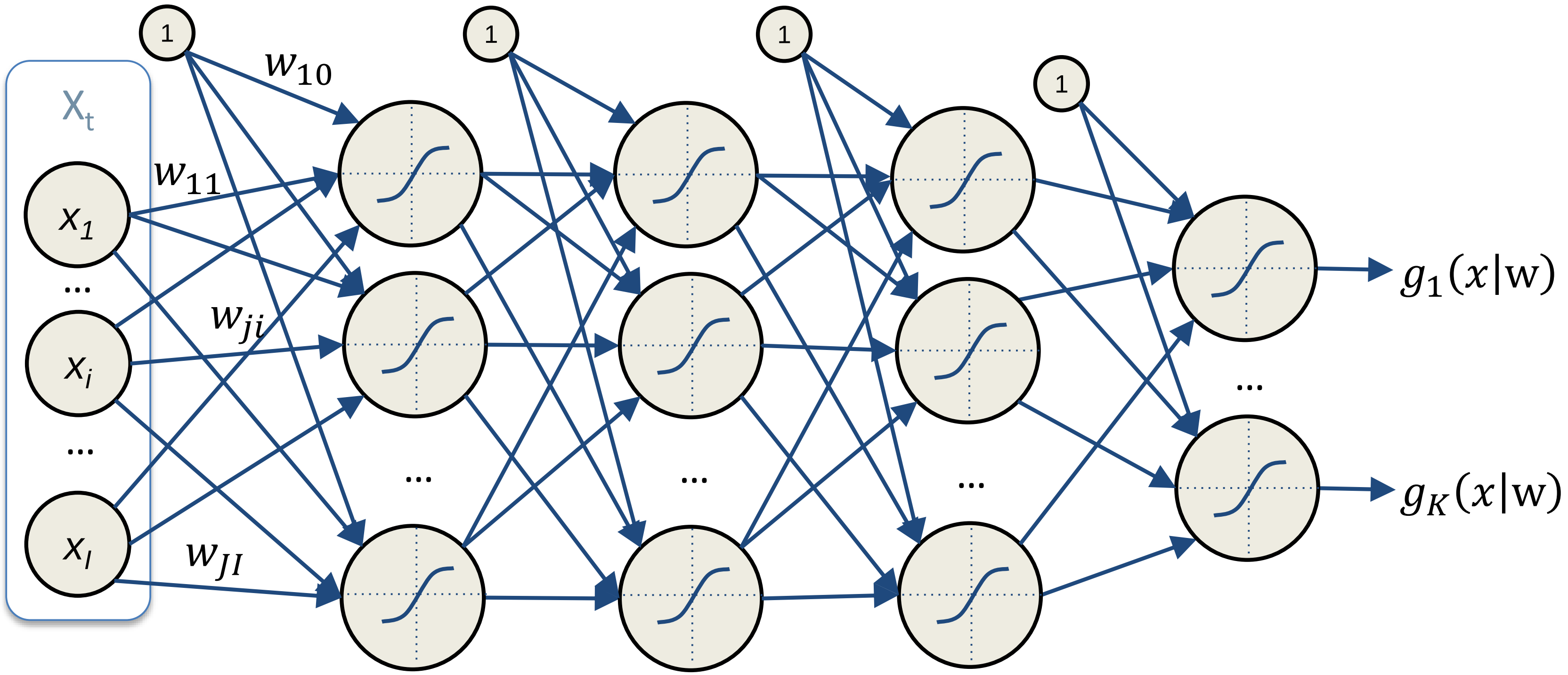
Sequence Modeling

So far we have considered only «static» datasets



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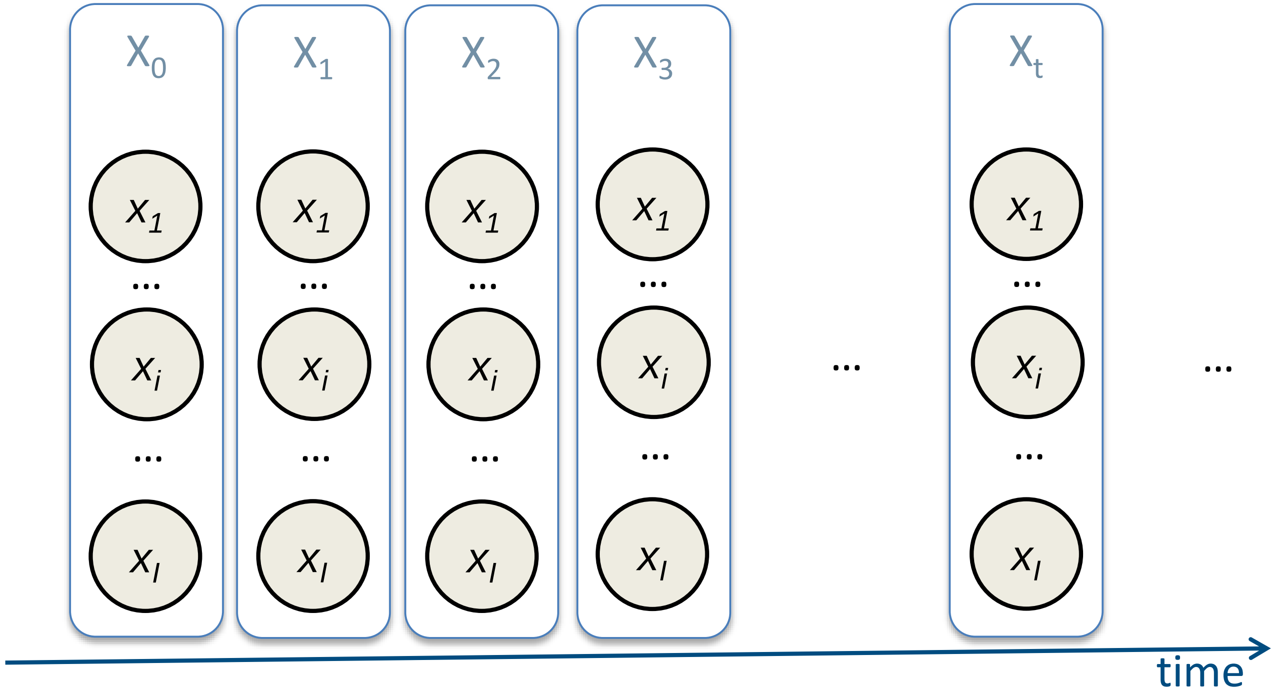


Sequence Modeling

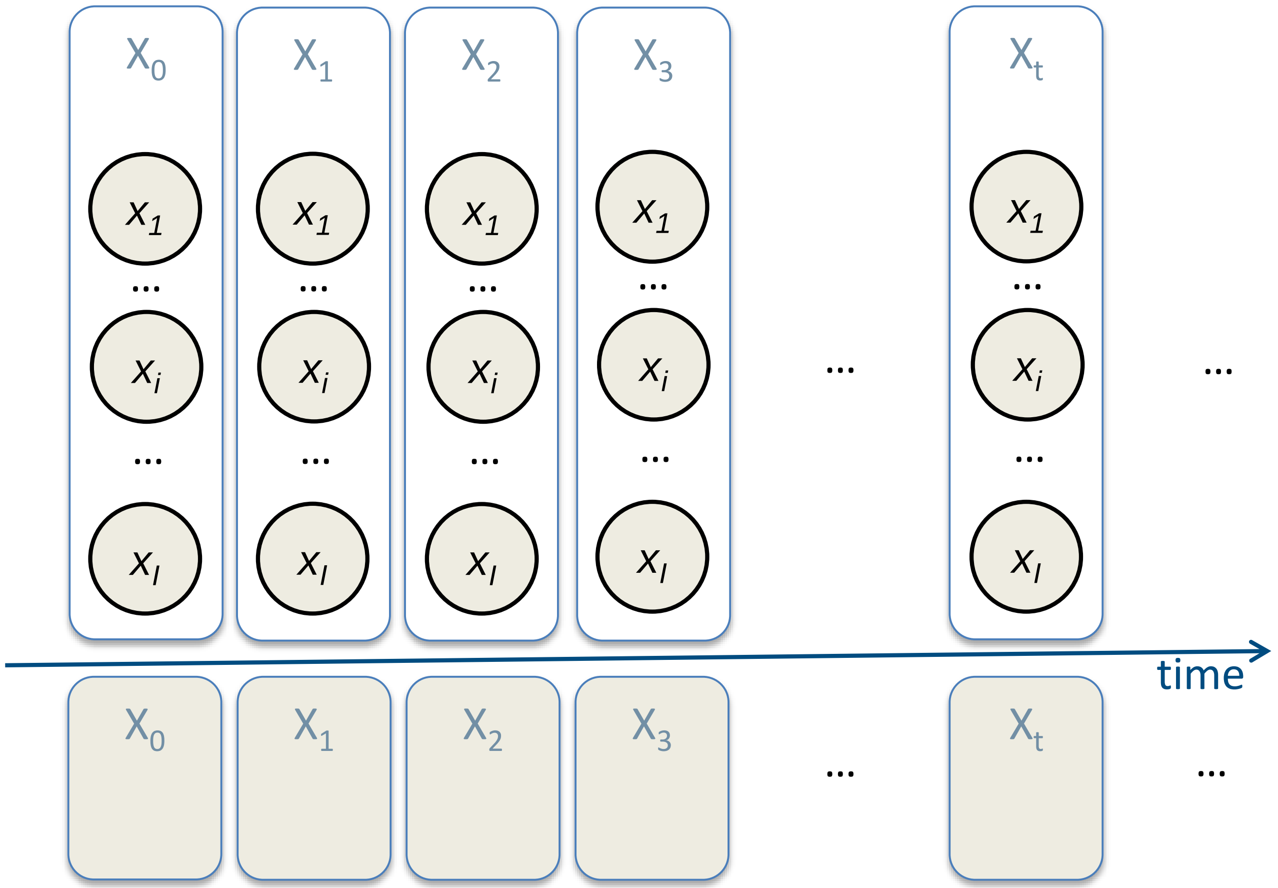
Different ways to deal with «dynamic» data:

Memoryless models:

- Autoregressive models
- Feedforward neural networks

Models with memory:

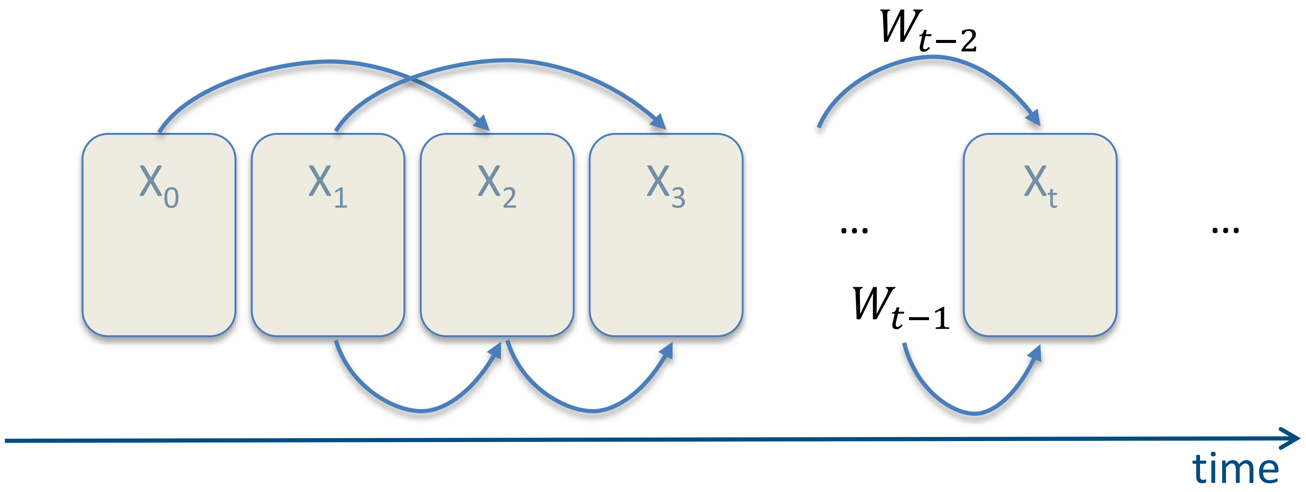
- Linear dynamical systems
- Hidden Markov models
- Recurrent Neural Networks
- …



Memoryless Models for Sequences

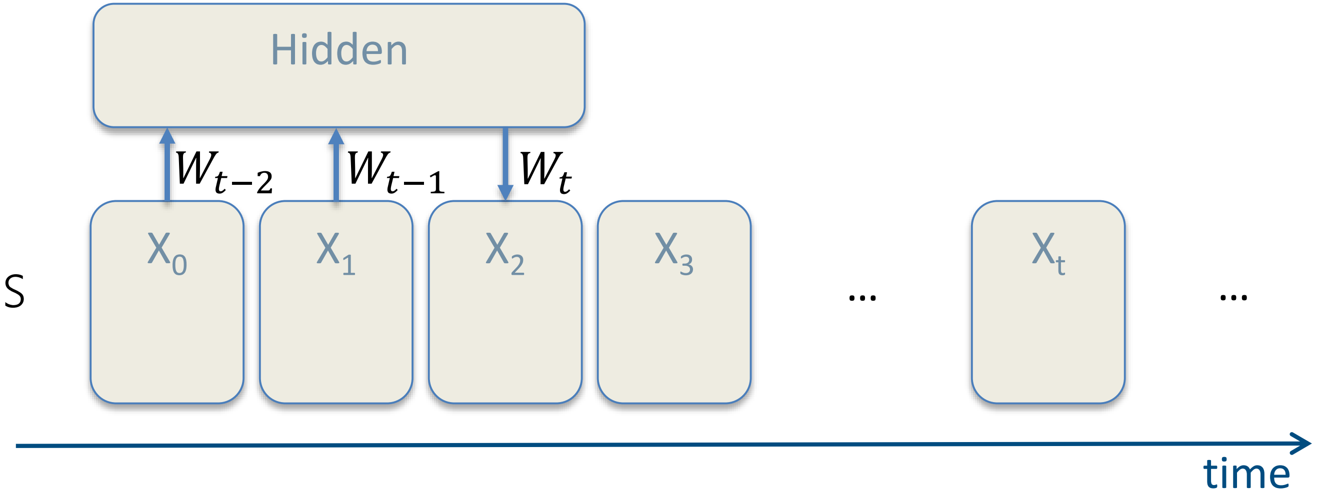
Autoregressive models

- Predict the next input from previous ones using «delay taps»



Feed forward neural networks

- Generalize autoregressive models using non linear hidden layers



Dynamical Systems

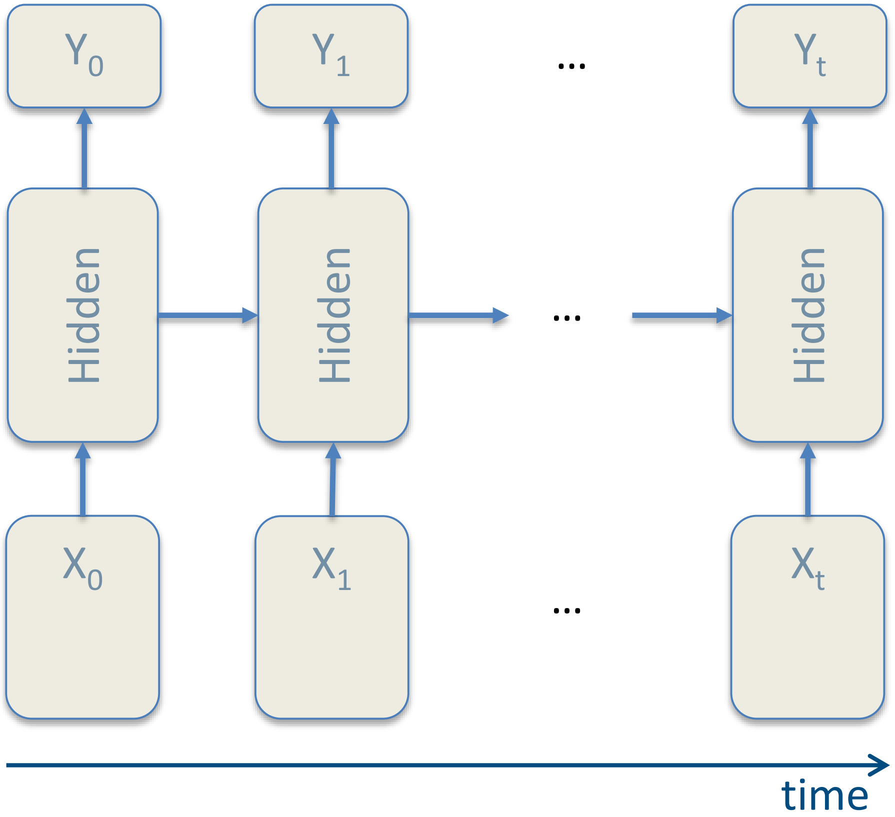
Stochastic systems …

Generative models with a real-valued hidden state which cannot be observed directly

- The hidden state has some dynamics possibly affected by noise and produces the output
- To compute the output has to infer hidden state
- Input are treated as driving inputs

In linear dynamical systems this becomes:

- State continuous with Gaussian uncertainty
- Transformations are assumed to be linear
- State can be estimated using *Kalman filtering*



Dynamical Systems

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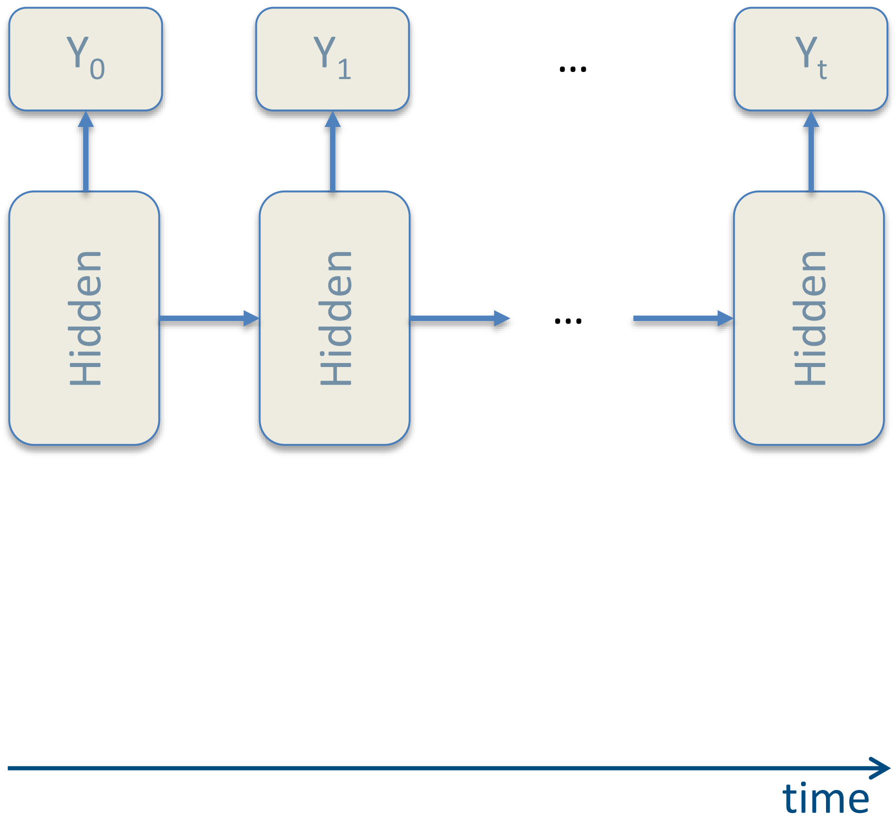


In hidden Markov models this becomes:

- State assumed to be discrete, state transitions are stochastic (transition matrix)
- Output is a stochastic function of hidden states
- State can be estimated via *Viterbi algorithm*.

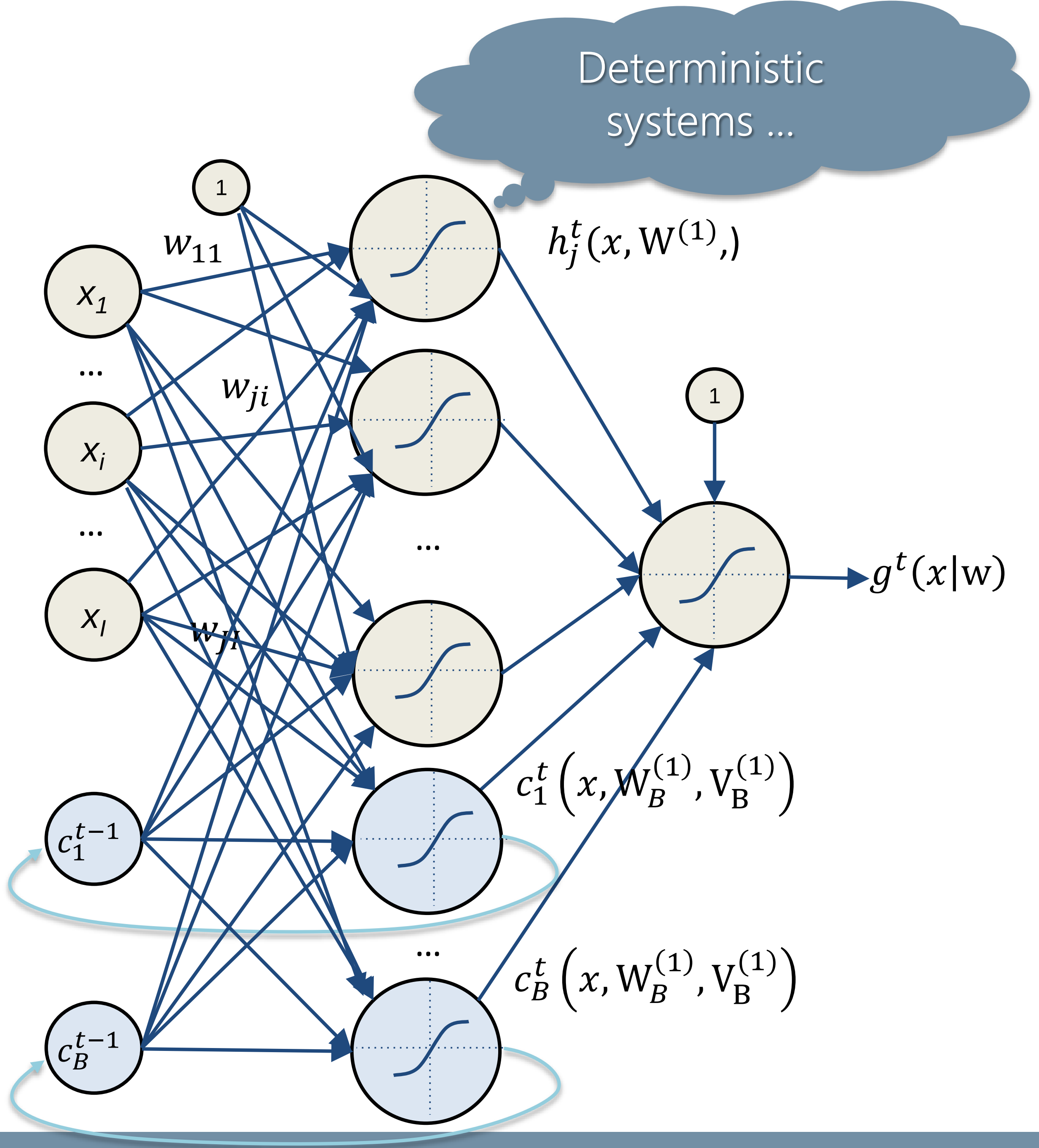
Recurrent Neural networks

Memory via recurrent connections:

- Distributed hidden state allows to store a information efficiently
- Non-linear dynamics allows complex hidden state updates

"With enough neurons and time, RNNs can compute anything that can be computed by a computer."

(Computation Beyond the Turing Limit
Hava T. Siegelmann, 1995)



Recurrent Neural networks

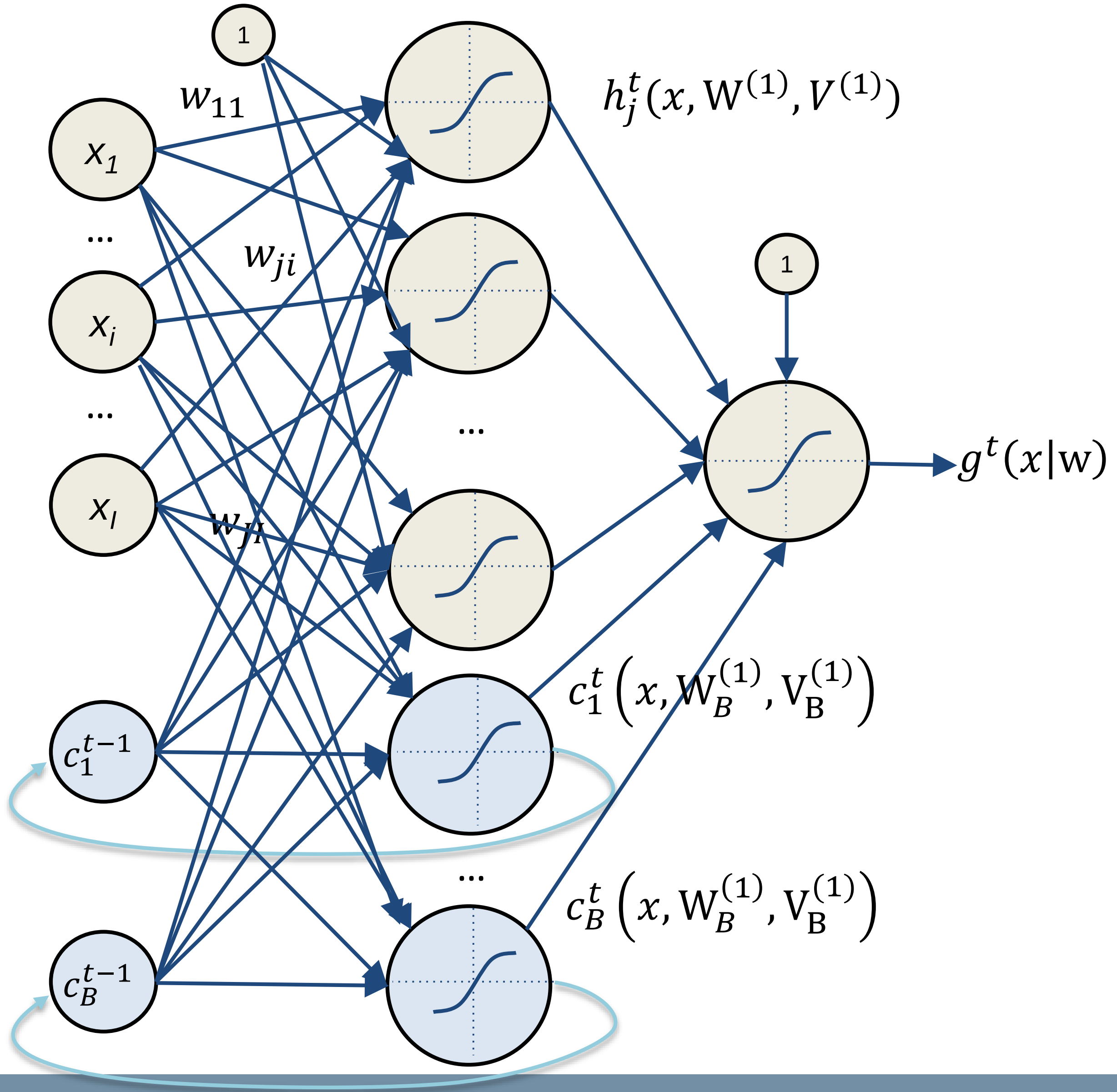
Memory via recurrent connections:

- Distributed hidden state allows to store a information efficiently
- Non-linear dynamics allows complex hidden state updates

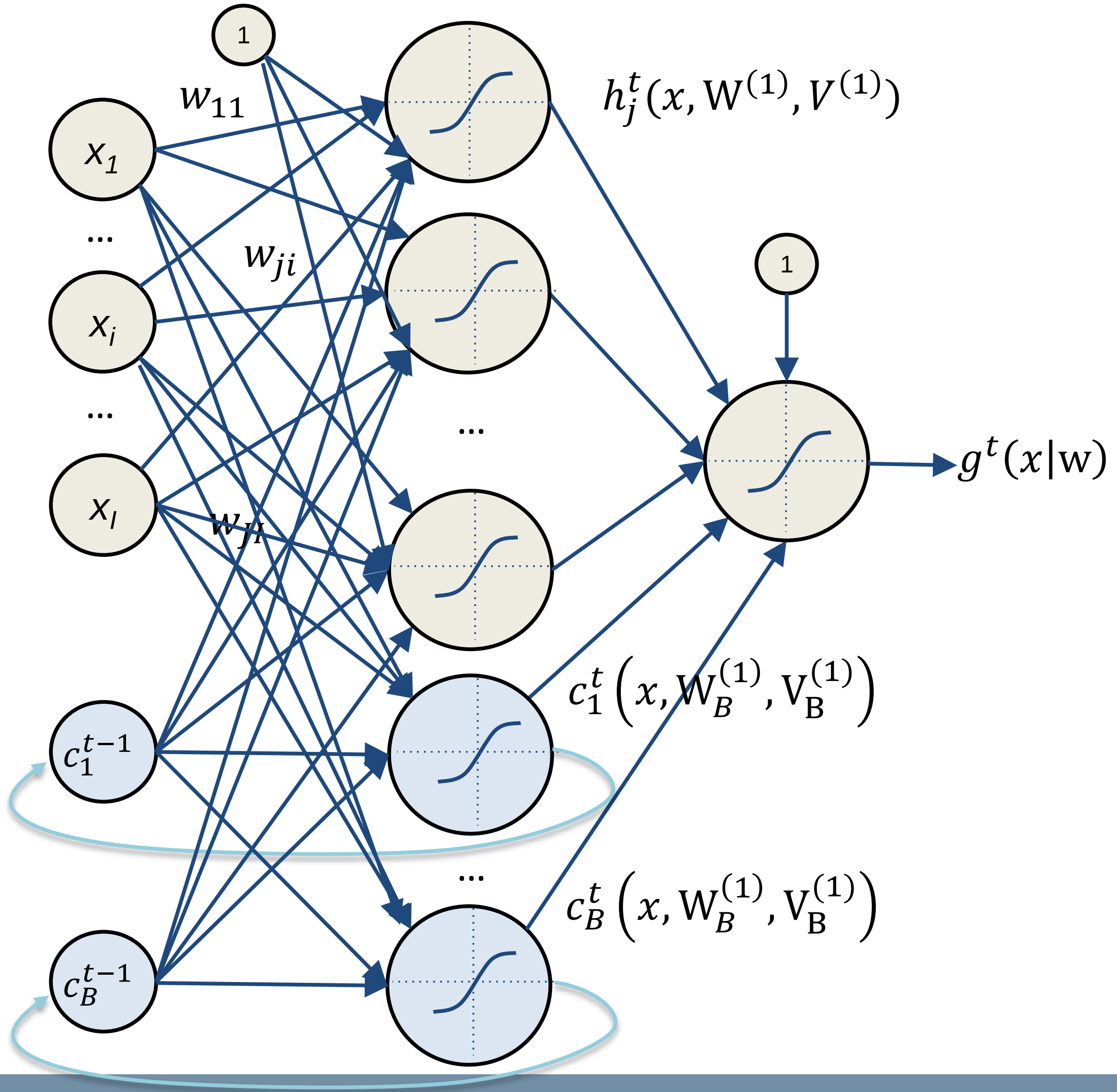
$$g^t(x_n|w) = g\left(\sum_{j=0}^{J} w_{1j}^{(2)} \cdot h_j^t(\cdot) + \sum_{b=0}^{B} v_{1b}^{(2)} \cdot c_b^t(\cdot)\right)$$

$$h_j^t(\cdot) = h_j^t\left(\sum_{j=0}^{J} w_{ji}^{(1)} \cdot x_{i,n} + \sum_{b=0}^{B} v_{jb}^{(1)} \cdot c_b^{t-1}\right)$$

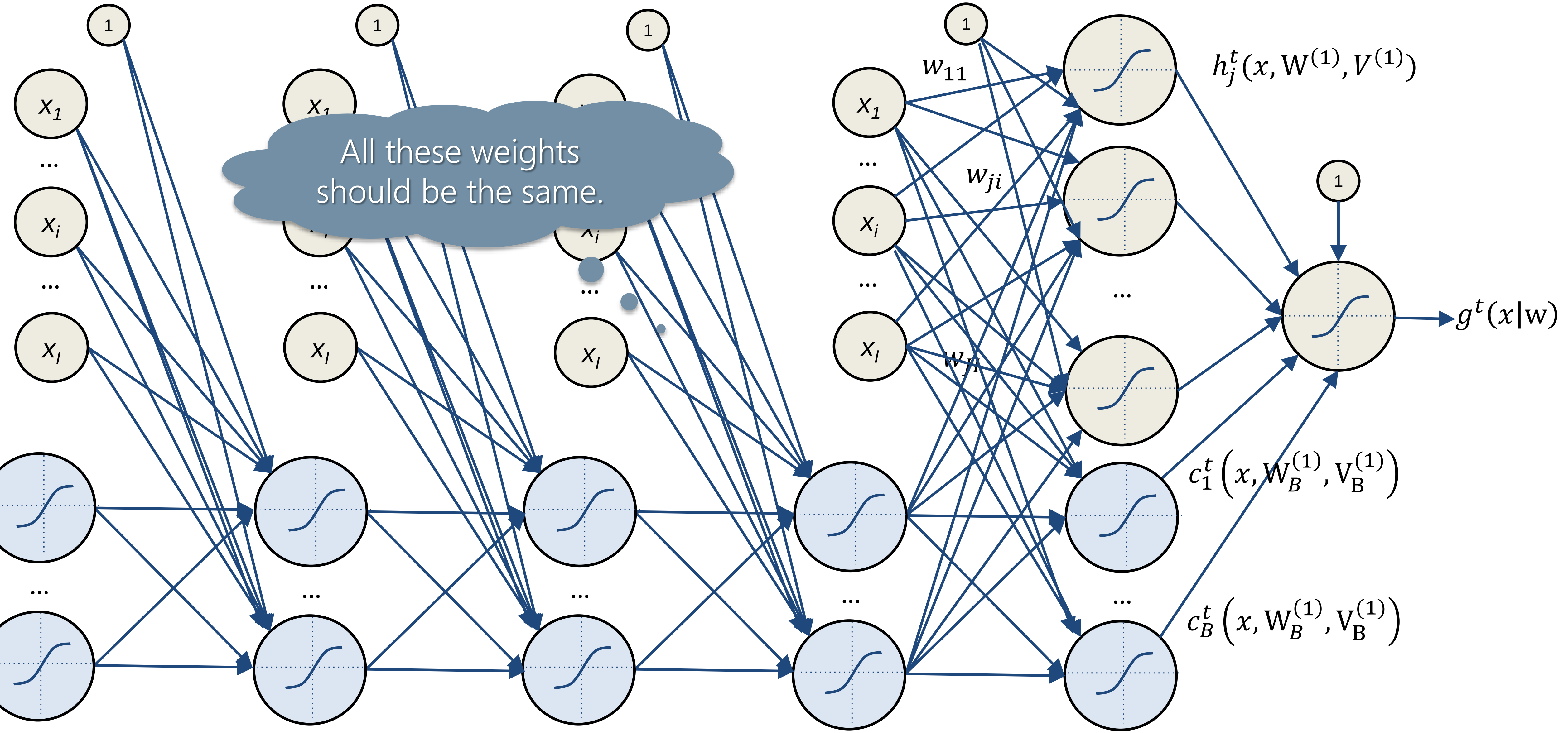
$$c_b^t(\cdot) = c_b^t\left(\sum_{j=0}^{J} v_{bi}^{(1)} \cdot x_{i,n} + \sum_{b\prime=0}^{B} v_{bb\prime}^{(1)} \cdot c_{b\prime}^{t-1}\right)$$



Backpropagation Through Time



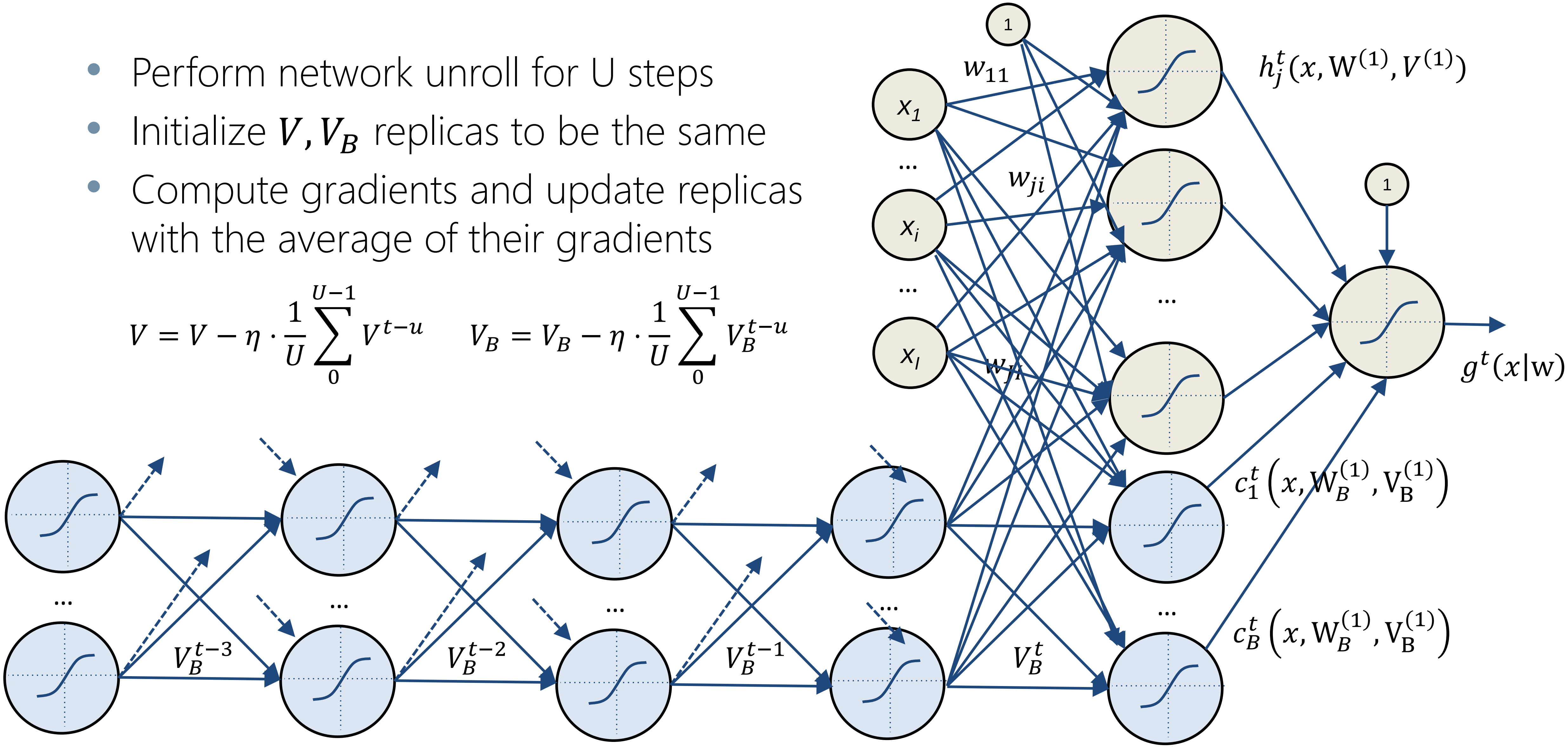
Backpropagation Through Time



Backpropagation Through Time

- Perform network unroll for U steps
- Initialize V, V_B replicas to be the same
- Compute gradients and update replicas with the average of their gradients

$$V = V - \eta \cdot \frac{1}{U} \sum_{0}^{U-1} V^{t-u} \qquad V_B = V_B - \eta \cdot \frac{1}{U} \sum_{0}^{U-1} V_B^{t-u}$$

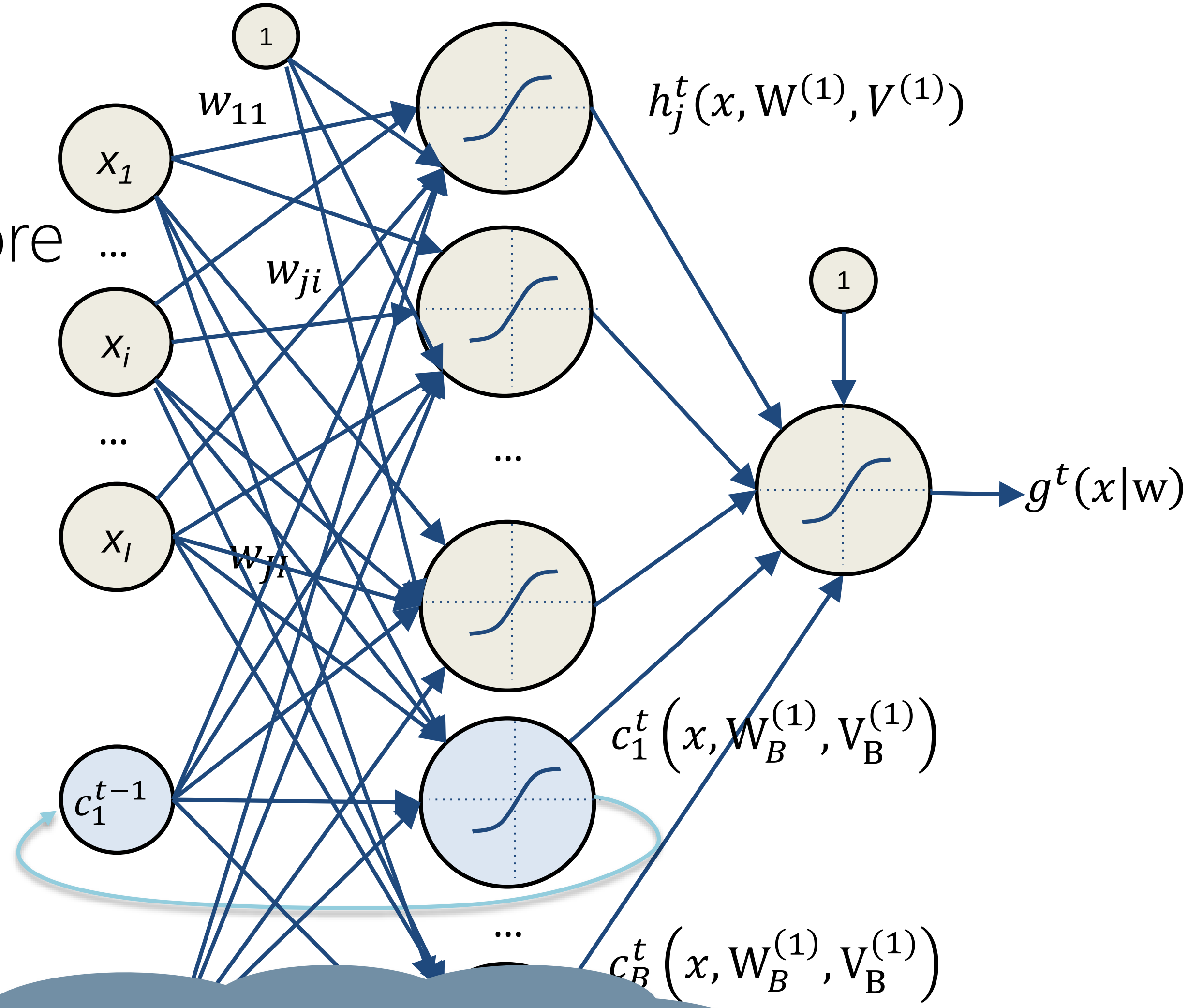


How much should we go back in time?

Sometime output might be related to some input happened quite long before

Jane walked into the room. John walked in too.
It was late in the day. Jane said hi to <???>

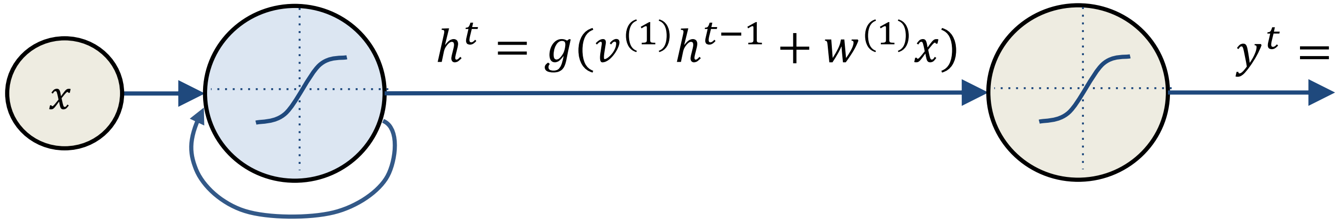
However backpropagation through time was not able to train recurrent neural networks significantly back in time …



Was due to not being able to backprop through many layers …

How much can we go back in time?

To better understand why it was not working consider a simplified case:



$h^t = g(v^{(1)}h^{t-1} + w^{(1)}x)$

$y^t = w^{(2)}g(h^t)$

Backpropagation over an entire sequence is computed as

$$\frac{\partial E}{\partial w} = \sum_{t=1}^{S} \frac{\partial E^t}{\partial w} \Rightarrow \frac{\partial E^t}{\partial w} = \sum_{t=1}^{t} \frac{\partial E^t}{\partial y^t}\frac{\partial y^t}{\partial h^t}\frac{\partial h^t}{\partial h^k}\frac{\partial h^k}{\partial w} \Rightarrow \frac{\partial h^t}{\partial h^k} = \prod_{i=k+1}^{t} \frac{\partial h_i}{\partial h_{i-1}} = \prod_{i=k+1}^{t} v^{(1)} g'(h^{i-1})$$

If we consider the norm of these terms

$$\left\|\frac{\partial h_i}{\partial h_{i-1}}\right\| = \|v^{(1)}\|\|g'(h^{i-1})\| \Rightarrow \left\|\frac{\partial h^t}{\partial h^k}\right\| \le (\gamma_v \cdot \gamma_{g'})^{t-k}$$

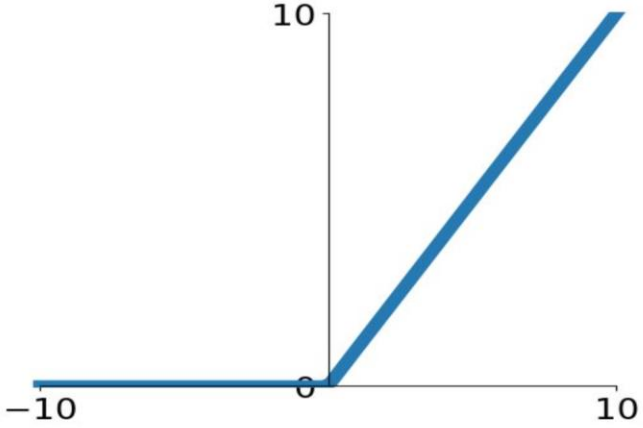
If $(\gamma_v \gamma_g') < 1$ this converges to 0 …

With Sigmoids and Tanh we have vanishing gradients

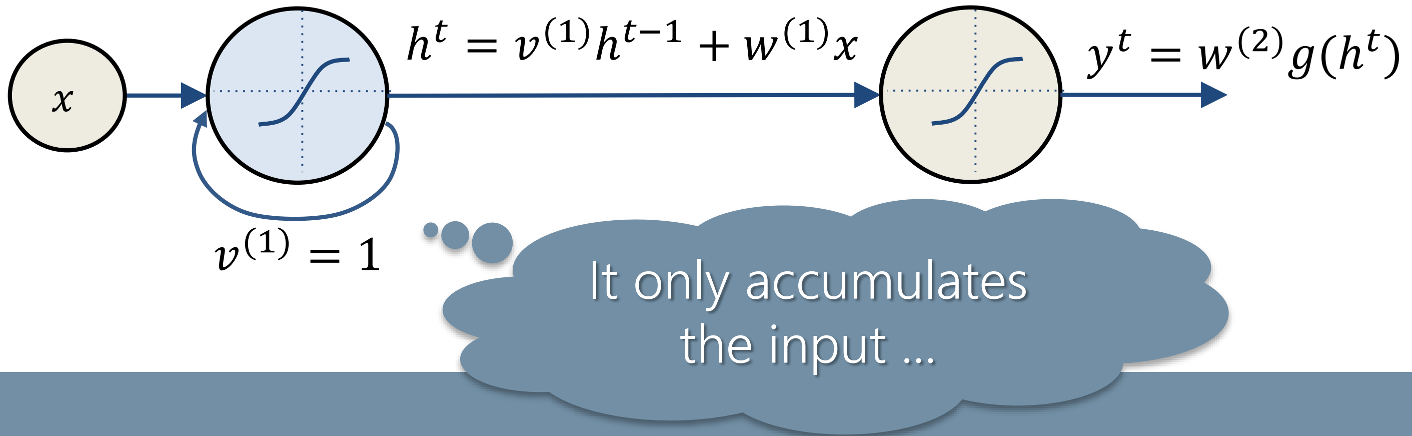
Dealing with Vanishing Gradient

Force all gradients to be either 0 or 1

$$g(a) = ReLu(a) = \max(0, a)$$
$$g'^{(a)} = 1_{a>0}$$



Build Recurrent Neural Networks using small modules that are designed to remember values for a long time.



Long-Short Term Memories

Hochreiter & Schmidhuber (1997) solved the problem of vanishing gradient designing a memory cell using logistic and linear units with multiplicative interactions:

- Information gets into the cell whenever its “*write*” gate is on.
- The information stays in the cell so long as its “*keep*” gate is on.
- Information is read from the cell by turning on its “*read*” gate.

Can backpropagate through this since the loop has fixed weight.

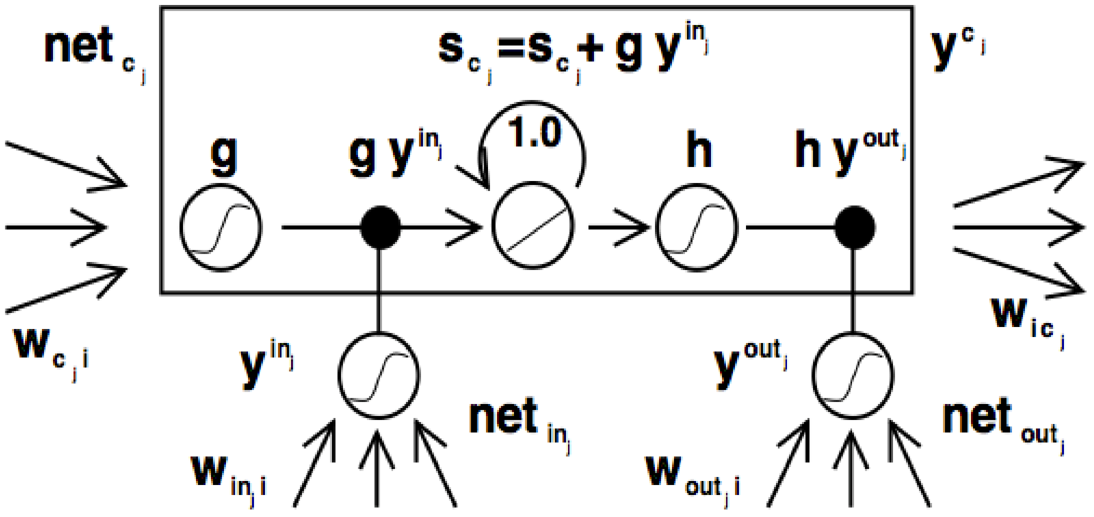
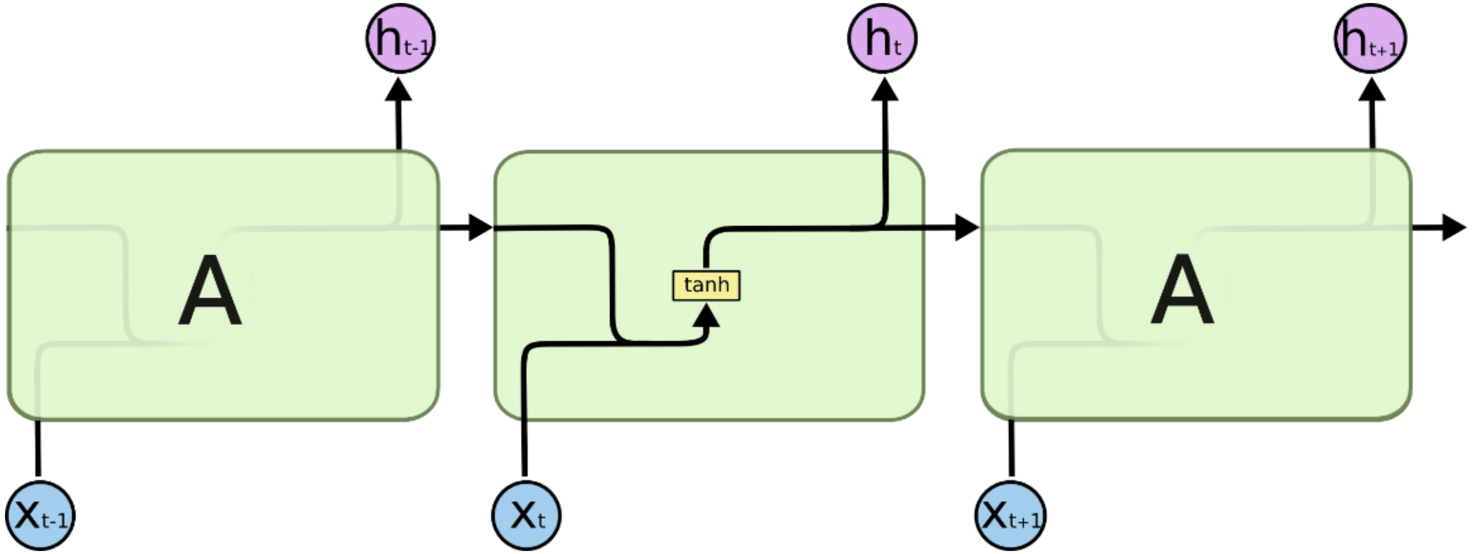


Figure 1: *Architecture of memory cell* c_j *(the box) and its gate units* in_j, out_j*. The self-recurrent connection (with weight 1.0) indicates feedback with a delay of 1 time step. It builds the basis of the “constant error carrousel” CEC. The gate units open and close access to CEC. See text and appendix A.1 for details.*

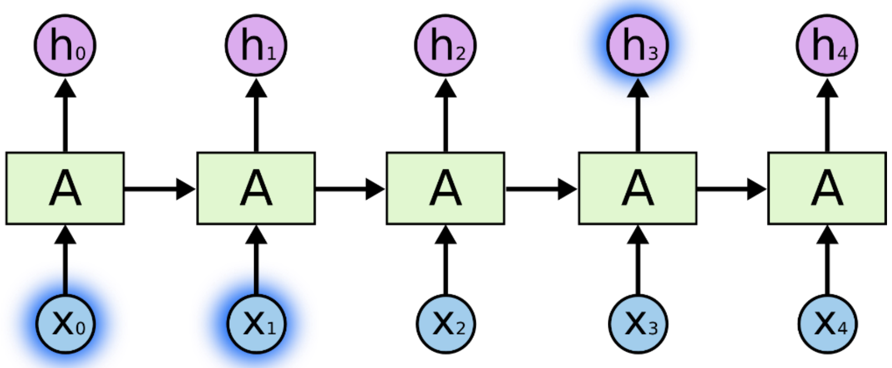
RNN vs. LSTM

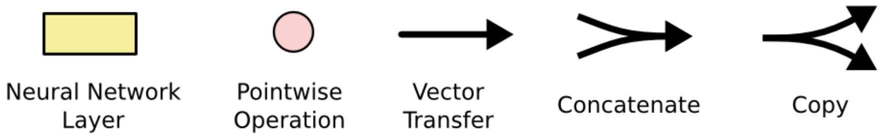
RNN



$$a_h^t = \sum_{i=1}^{I} w_{ih} x_i^t + \sum_{h'=1}^{H} w_{h'h} b_{h'}^{t-1}$$

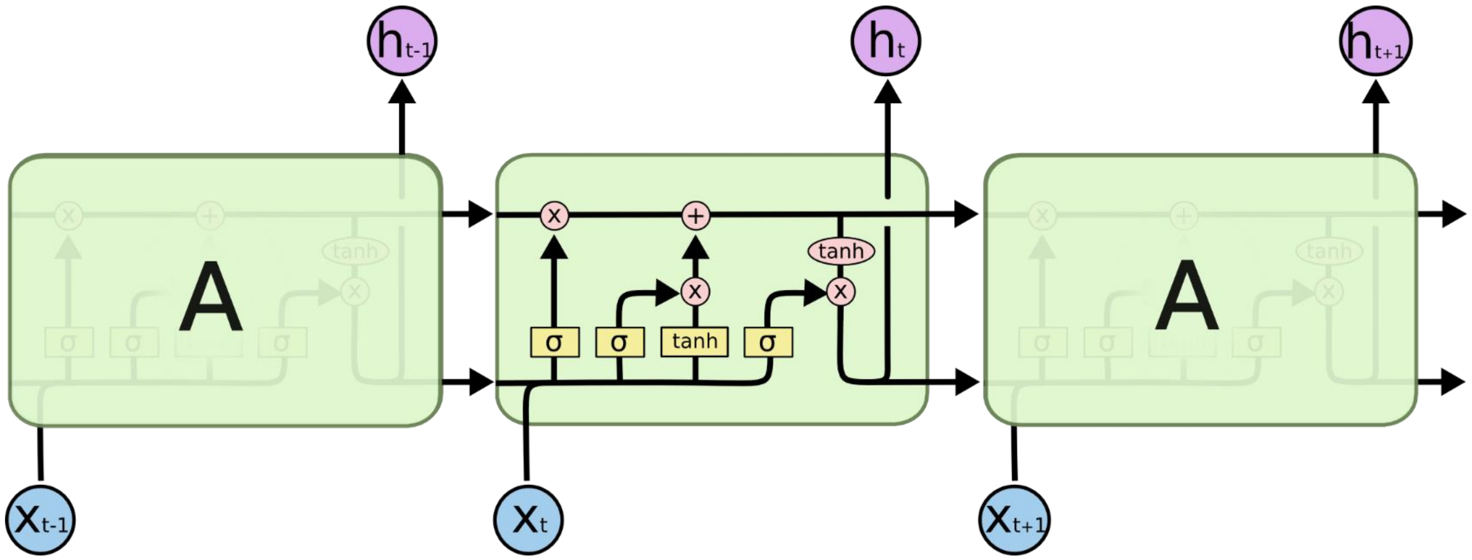
$$b_h^t = \theta_h(a_h^t)$$





Long Short Term Memory

LSTM



$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \;+\; b_i\right)$$

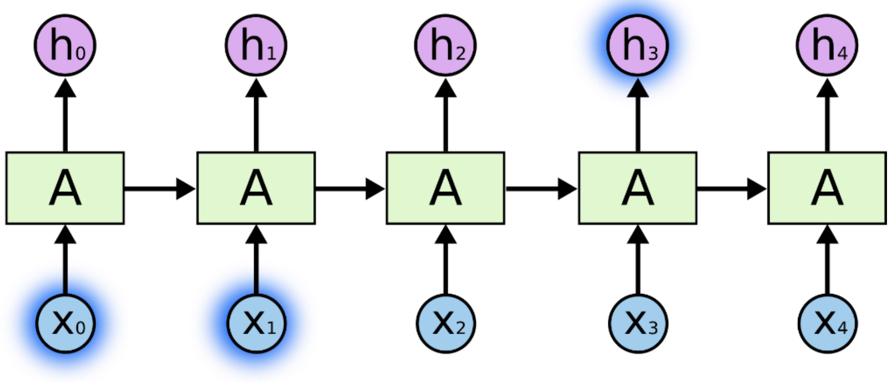
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \;+\; b_C)$$

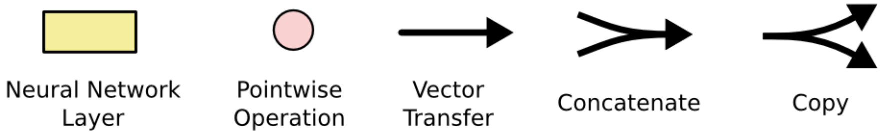
$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] \;+\; b_f\right)$$

$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

$$o_t = \sigma\left(W_o \; [h_{t-1}, x_t] \;+\; b_o\right)$$

$$h_t = o_t * \tanh\left(C_t\right)$$

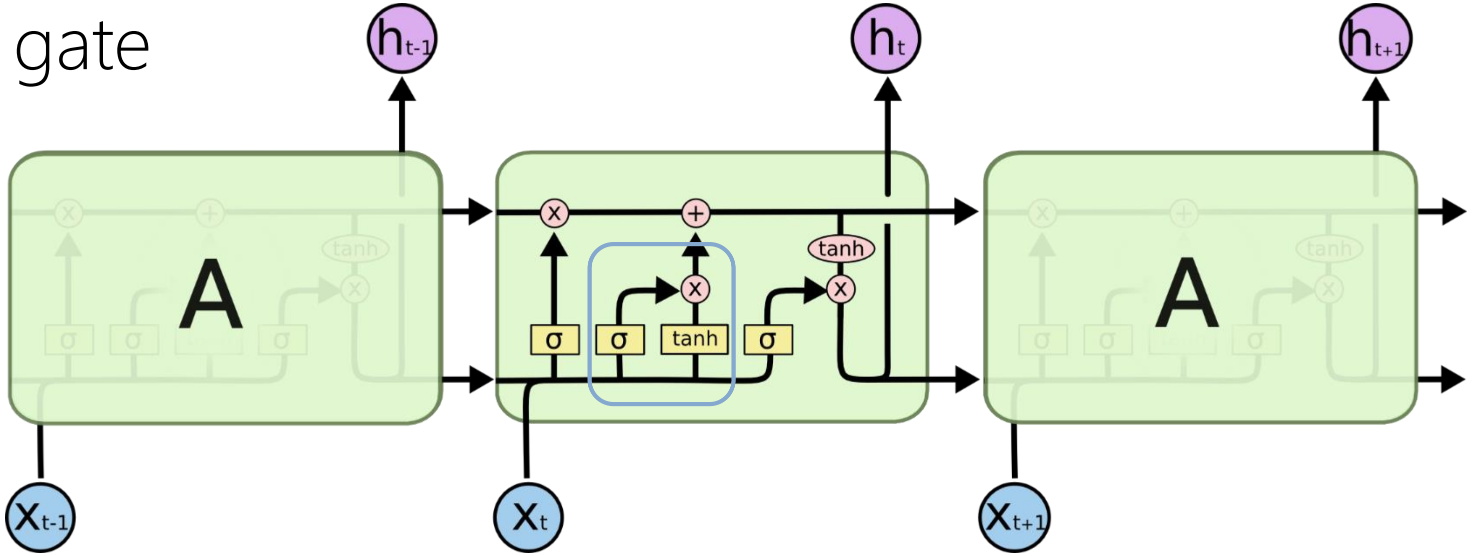




Long Short Term Memory

Input gate

LSTM



$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \;+\; b_i\right)$$
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \;+\; b_C)$$
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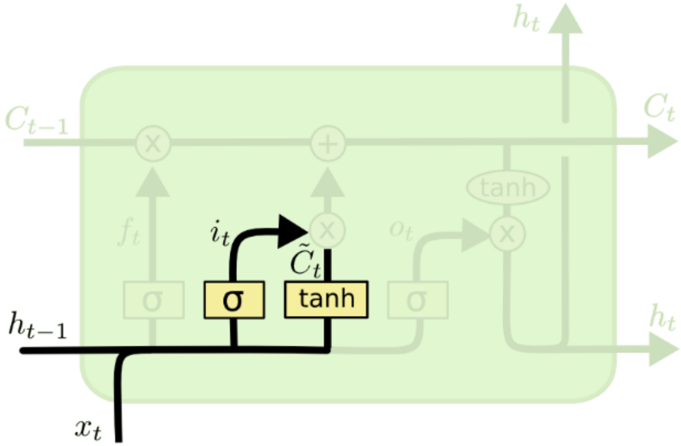
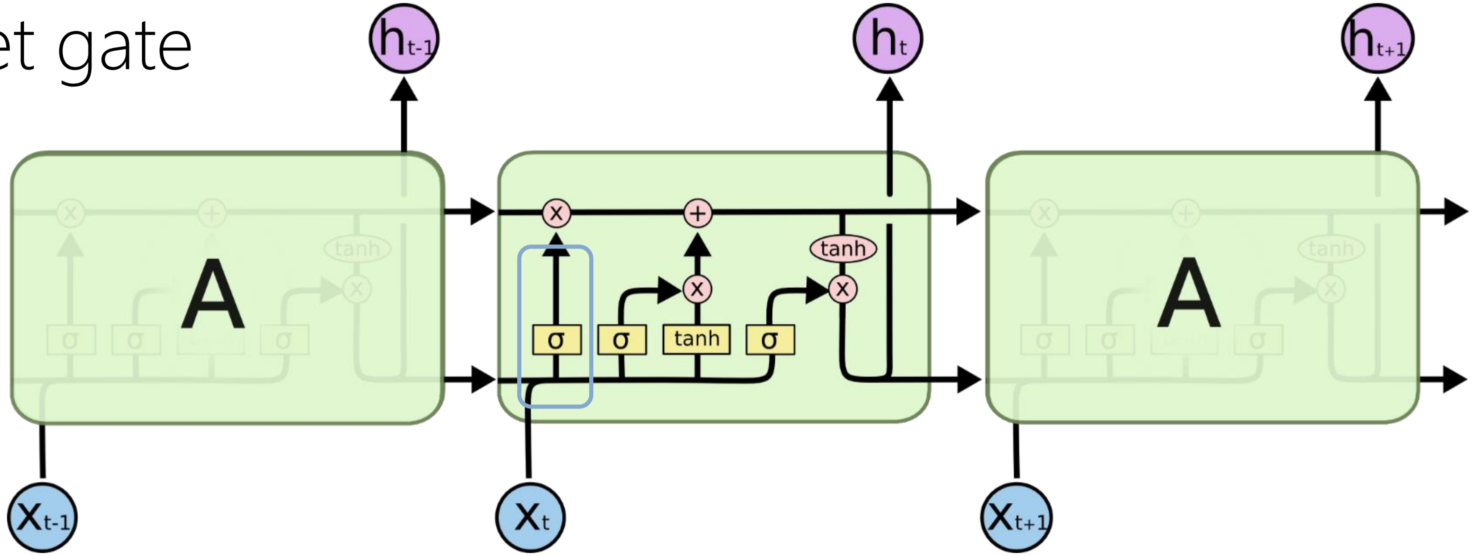


$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \;+\; b_i\right)$$
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \;+\; b_C)$$

Long Short Term Memory

Forget gate

LSTM



$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \; + \; b_i\right)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \; + \; b_C)$$

$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] \; + \; b_f\right)$$

$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

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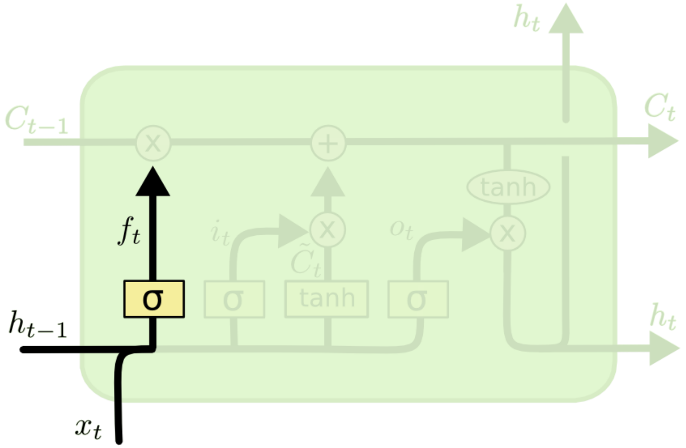
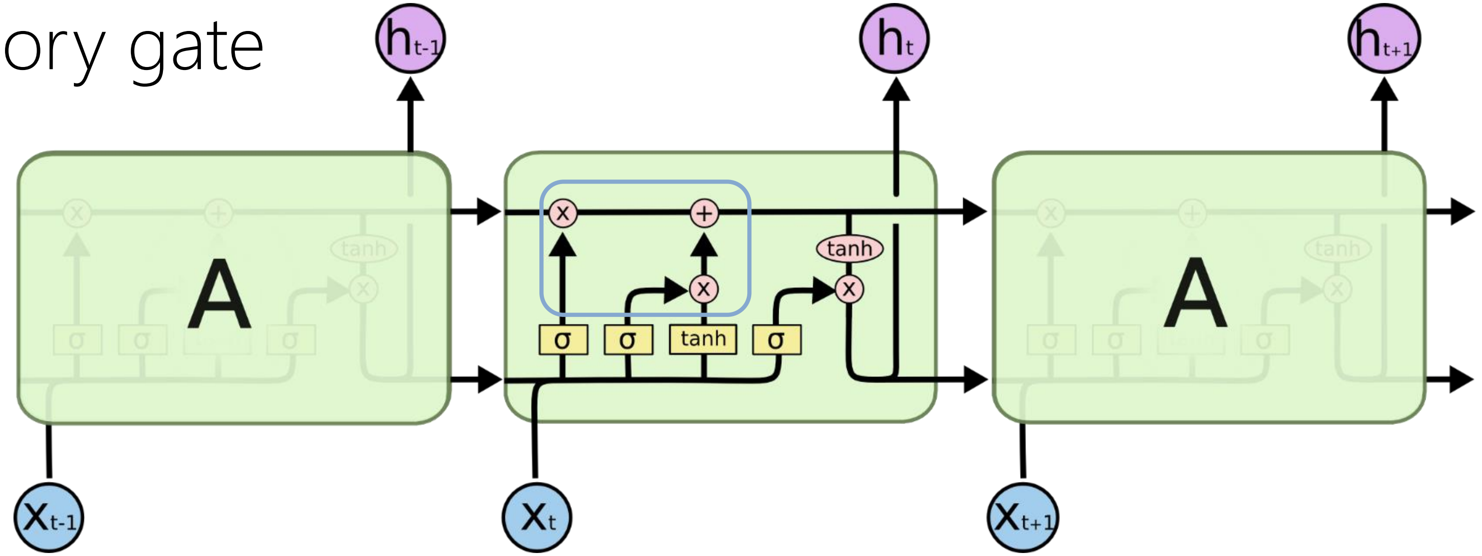


$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] \; + \; b_f\right)$$

Long Short Term Memory

Memory gate

LSTM



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$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \; + \; b_C)$$

$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] \; + \; b_f\right)$$

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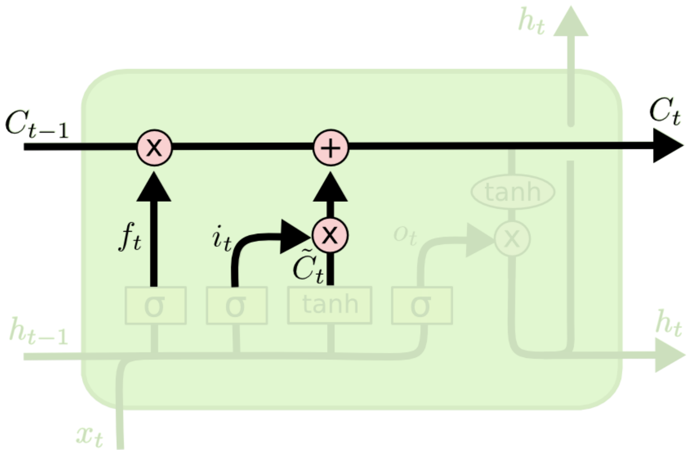
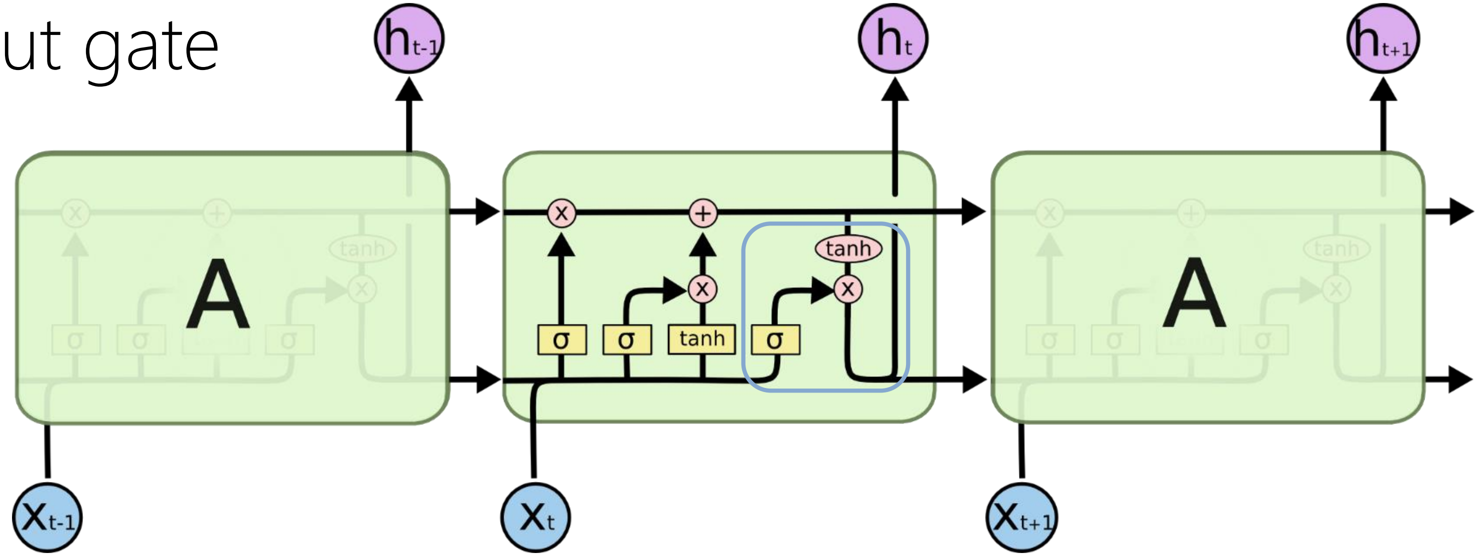


$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

Long Short Term Memory

Output gate

LSTM



$$i_t = \sigma\left(W_i\cdot[h_{t-1}, x_t] \; + \; b_i\right)$$
$$\tilde{C}_t = \tanh(W_C\cdot[h_{t-1}, x_t] \; + \; b_C)$$
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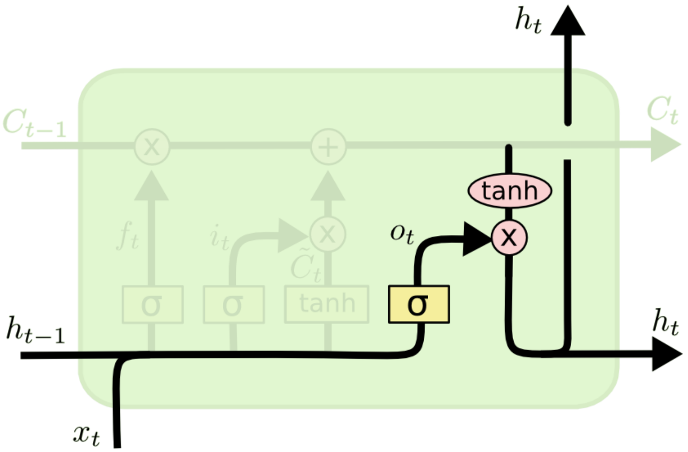
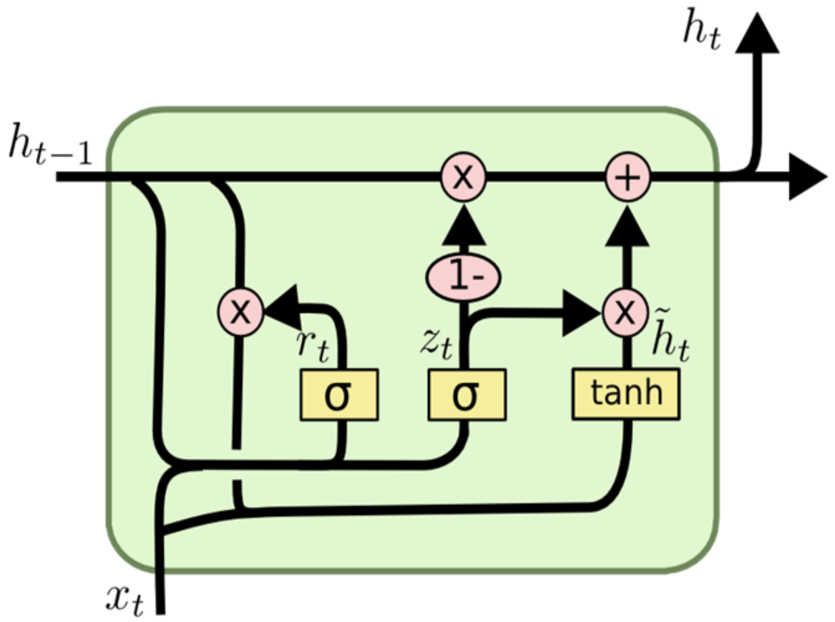


$$o_t = \sigma\left(W_o \; [h_{t-1}, x_t] \; + \; b_o\right)$$
$$h_t = o_t * \tanh\left(C_t\right)$$

Gated Recurrent Unit (GRU)

It combines the forget and input gates into a single "update gate." It also merges the cell state and hidden state, and makes some other changes.



$$z_t = \sigma\left(W_z \cdot [h_{t-1}, x_t]\right)$$

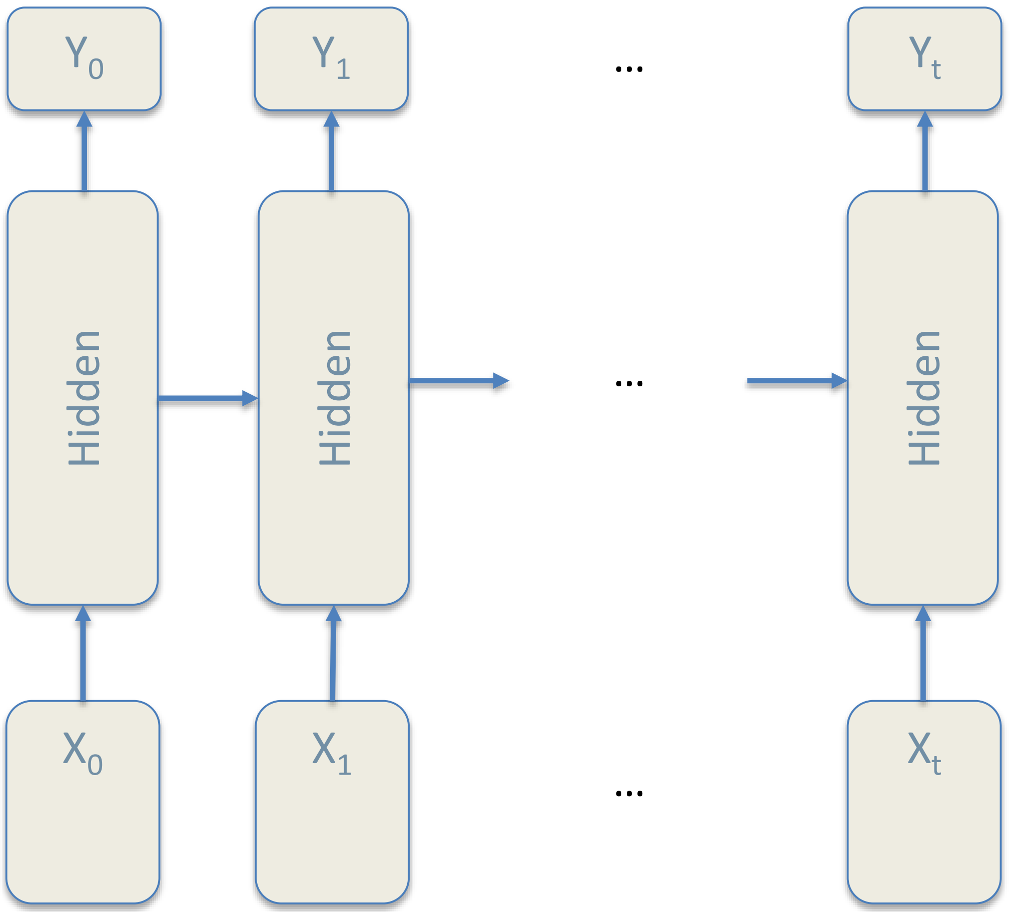
$$r_t = \sigma\left(W_r \cdot [h_{t-1}, x_t]\right)$$

$$\tilde{h}_t = \tanh\left(W \cdot [r_t * h_{t-1}, x_t]\right)$$

$$h_t = (1 - z_t) * h_{t-1} + z_t * \tilde{h}_t$$

LSTM Networks

You can build a computation graph with continuous transformations.



LSTM Networks

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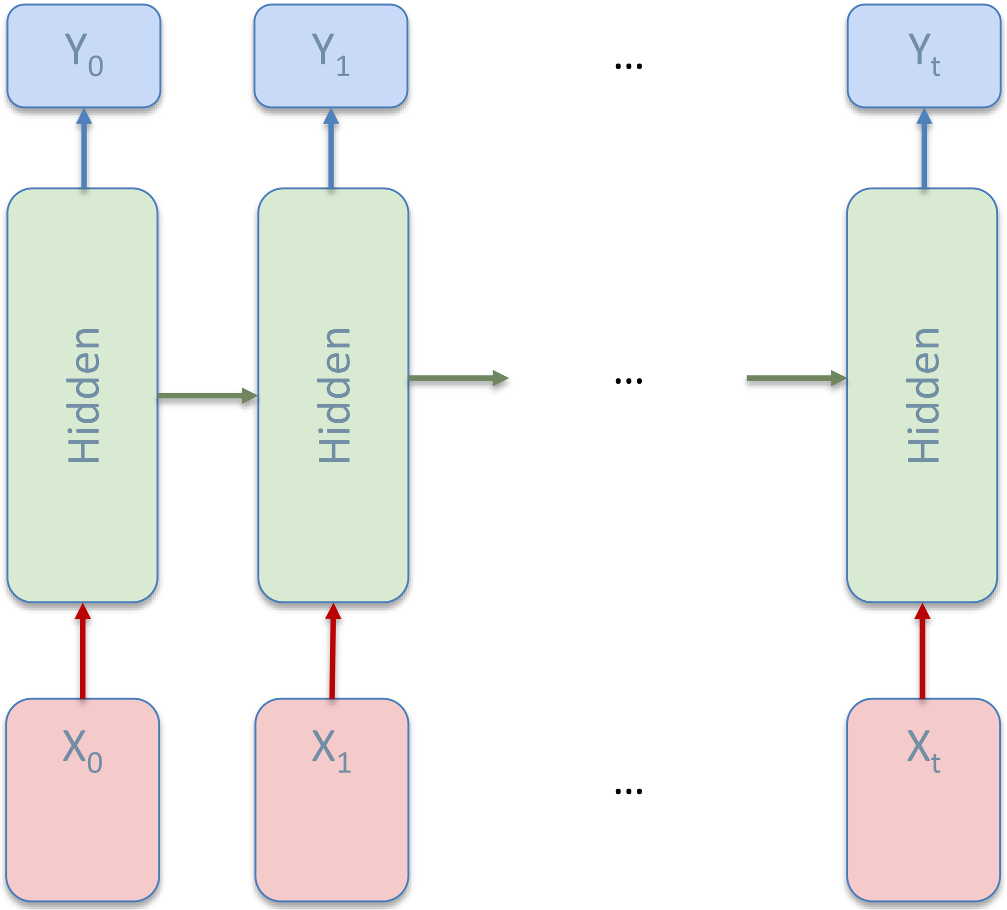

LSTM Networks

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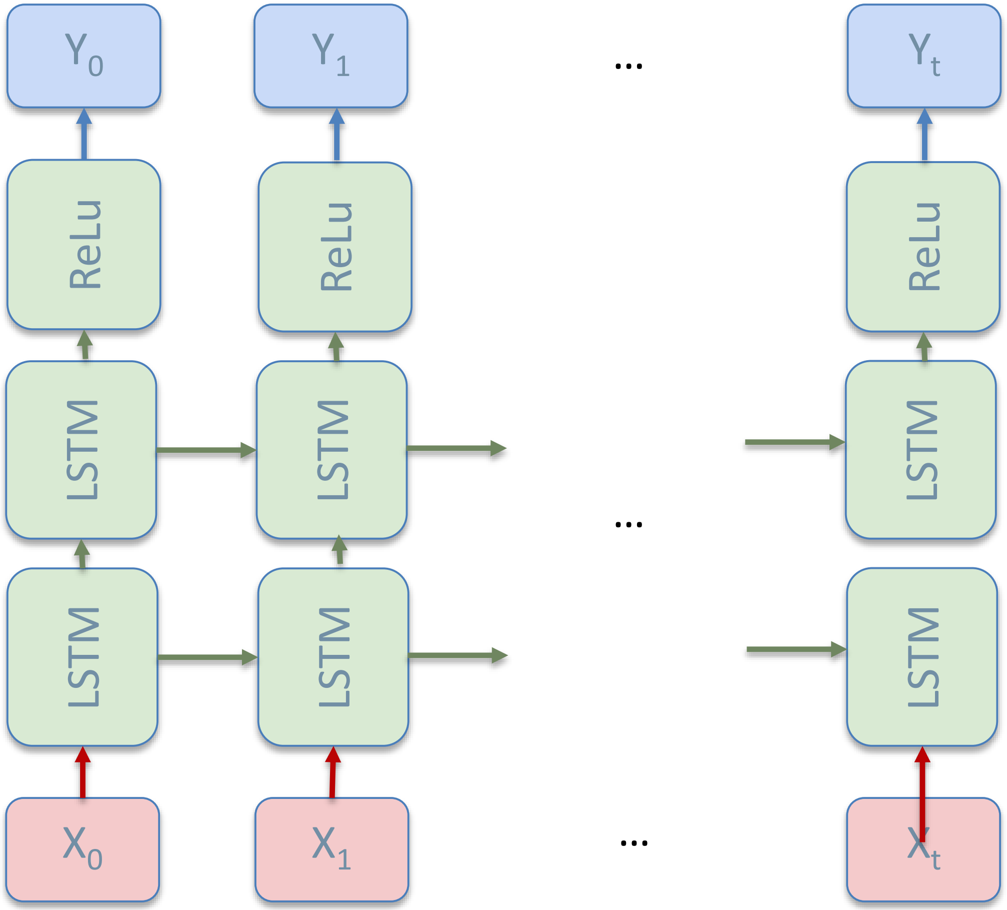
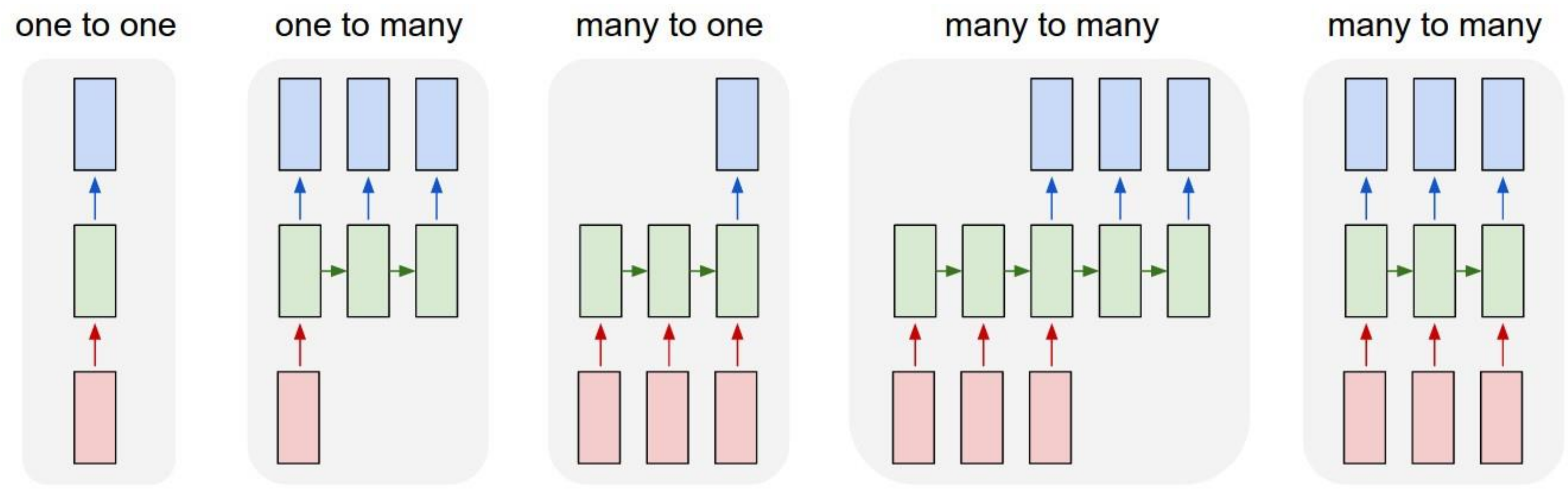


Sequential Data Problems



Fixed-sized input to fixed-sized output (e.g. image classification)

Sequence output (e.g. image captioning takes an image and outputs a sentence of words).

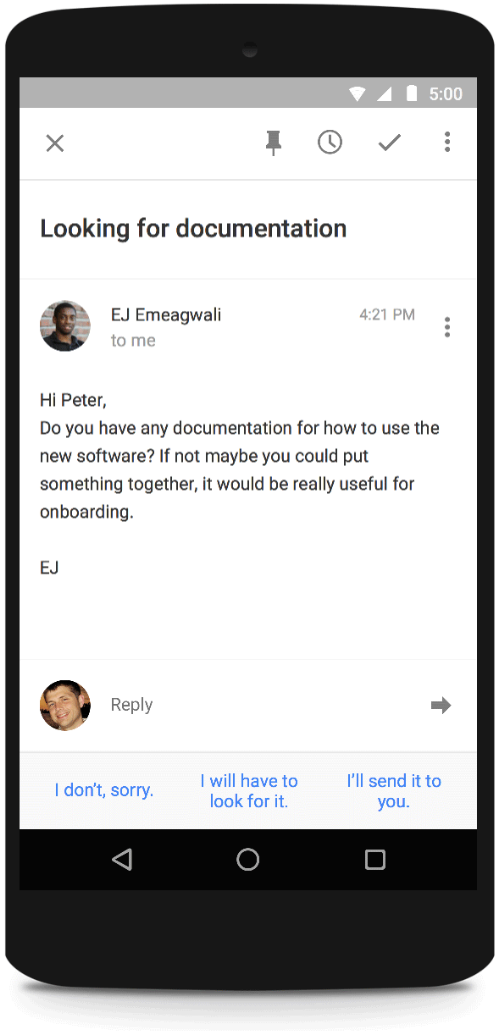
Sequence input (e.g. sentiment analysis where a given sentence is classified as expressing positive or negative sentiment).

Sequence input and sequence output (e.g. Machine Translation: an RNN reads a sentence in English and then outputs a sentence in French)

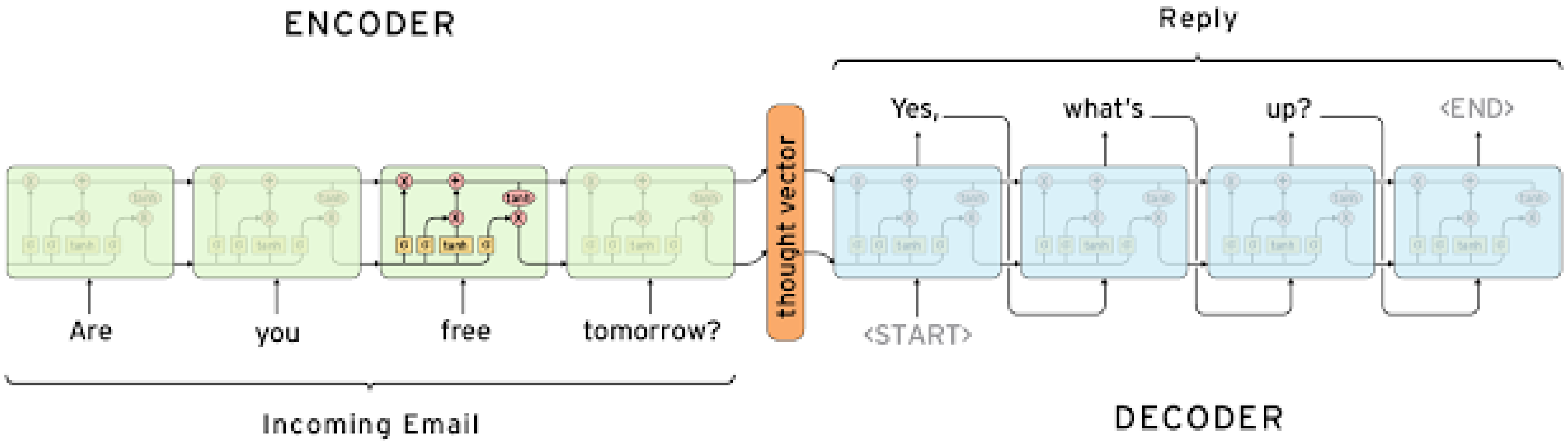
Synced sequence input and output (e.g. video classification where we wish to label each frame of the video)

Credits: Andrej Karpathy

Sequence to Sequence Modeling



Given <S, T> pairs, read S, and output T′ that best matches T



$$p(y_1, \ldots, y_{T'} | x_1, \ldots, x_T) = \prod_{t=1}^{T'} p(y_t | v, y_1, \ldots, y_{t-1})$$

$$1/|S| \sum_{(T,S)\in\mathcal{S}} \log p(T|S)$$

Tips & Tricks

When conditioning on full input sequence Bidirectional RNNs exploit it:

- Have one RNNs traverse the sequence left-to-right
- Have another RNN traverse the sequence right-to-left
- Use concatenation of hidden layers as feature representation

When initializing RNN we need to specify the initial state

- Could initialize them to a fixed value (such as 0)
- Better to treat the initial state as learned parameters
  - Start off with random guesses of the initial state values
  - Backpropagate the prediction error through time all the way to the initial state values and compute the gradient of the error with respect to these
  - Update these parameters by gradient descent