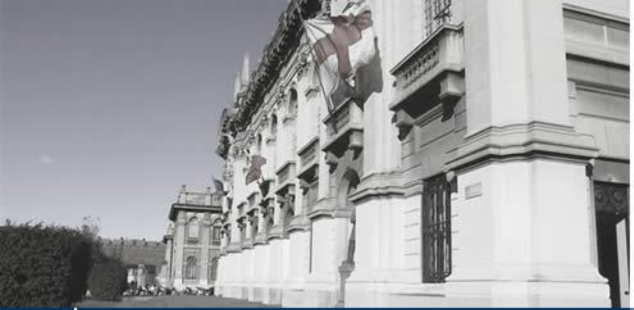
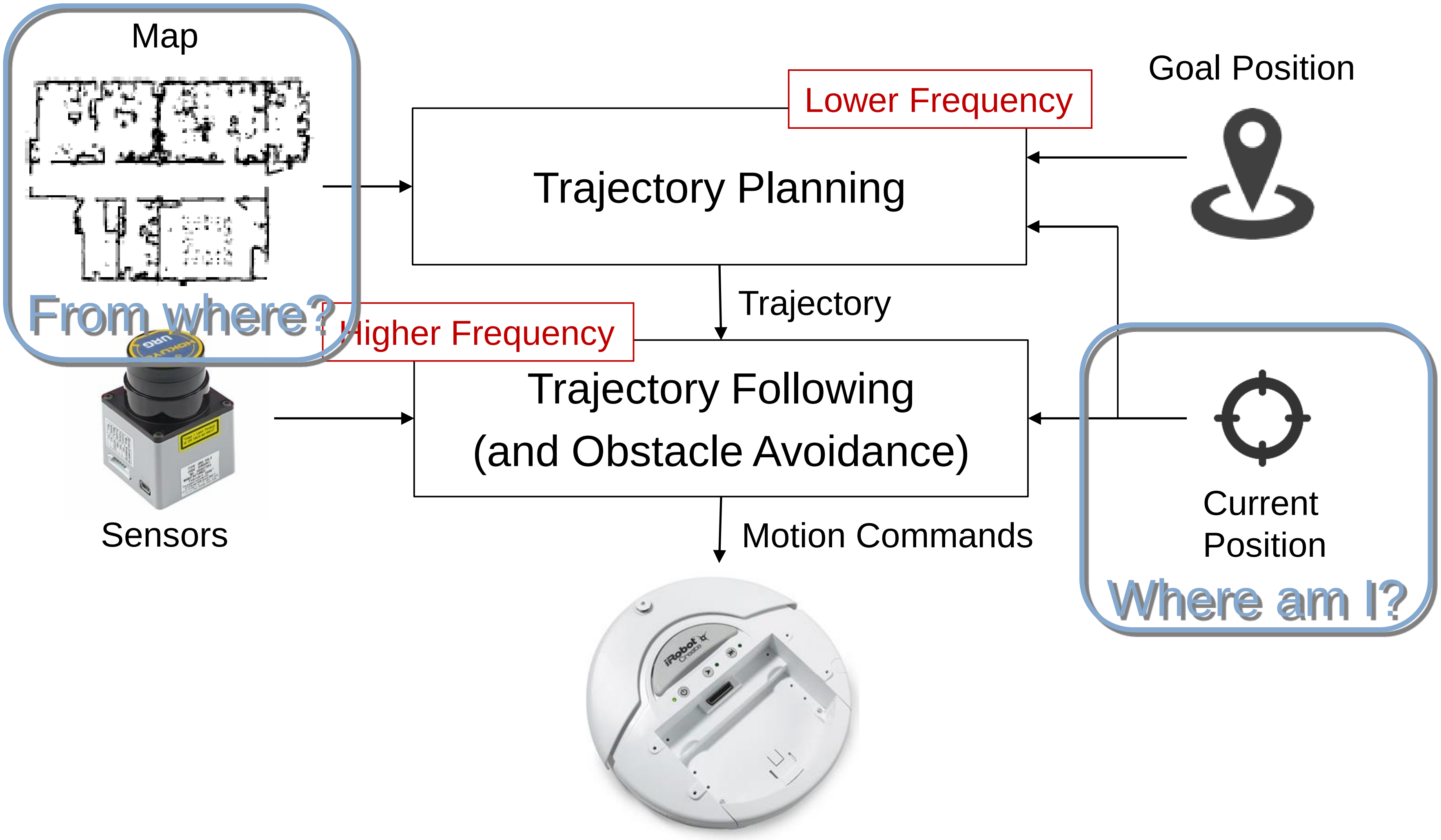


POLITECNICO DI MILANO

(Simultaneous) Localization & Mapping

Matteo Matteucci – matteo.matteucci@polimi.it

A Two Layered Approach



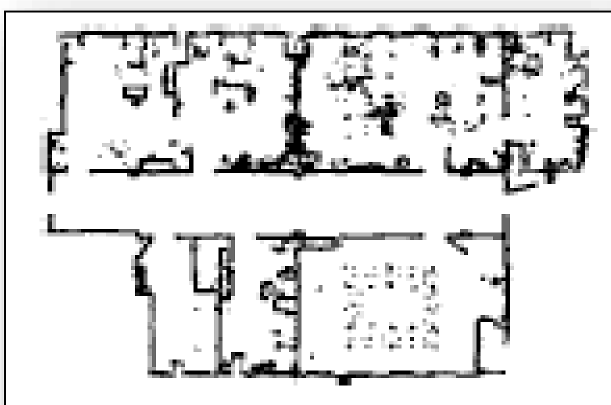
Where Am I?

To perform their tasks autonomous robots and unmanned vehicles need

- To know where they are (e.g., Global Positioning System)
- To know the environment map (e.g., Geographical Institutes Maps)

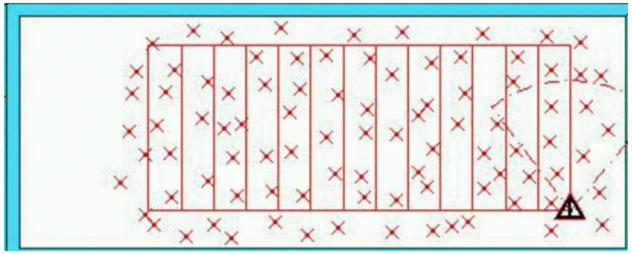
These are not always possible or reliable

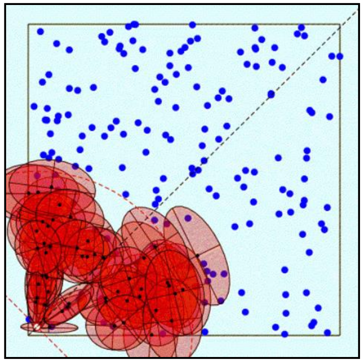
- GNSS are not always reliable/available
- Not all places have been mapped
- Environment changes dynamically
- Maps need to be updated

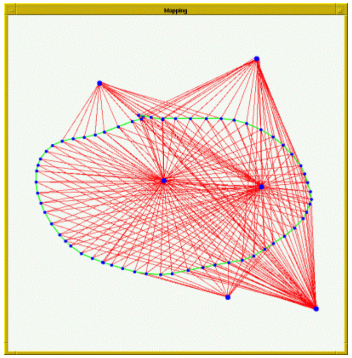


Representations

Landmark-based

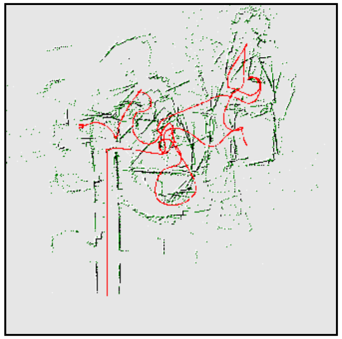


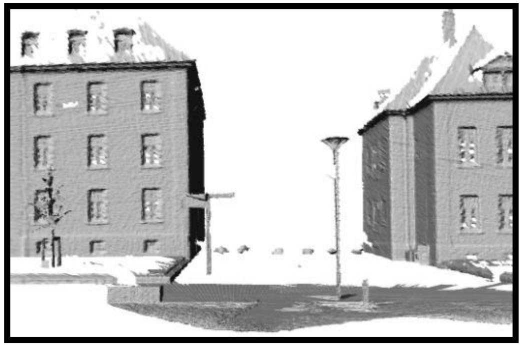


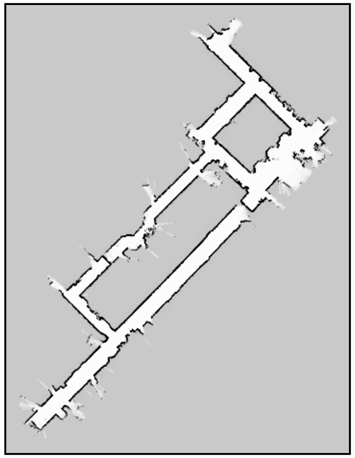


[Leonard et al., 98; Castelanos et al., 99: Dissanayake et al., 2001; Montemerlo et al., 2002;…]

Grid maps or scans

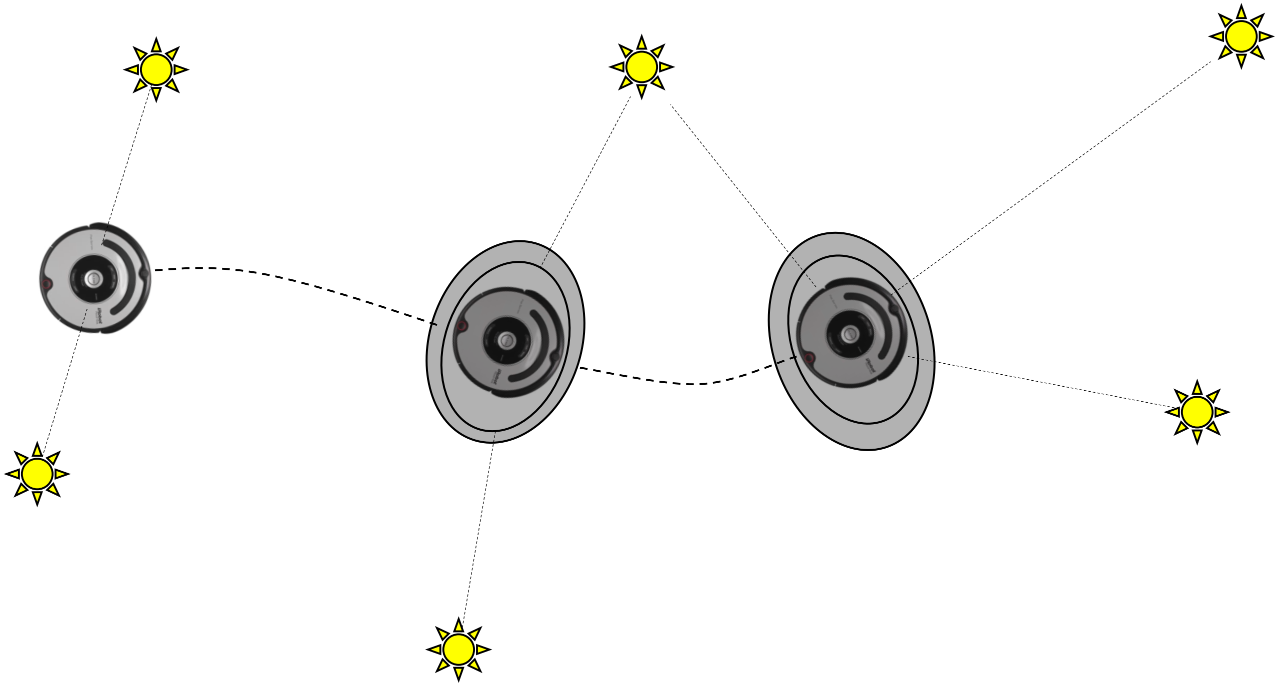




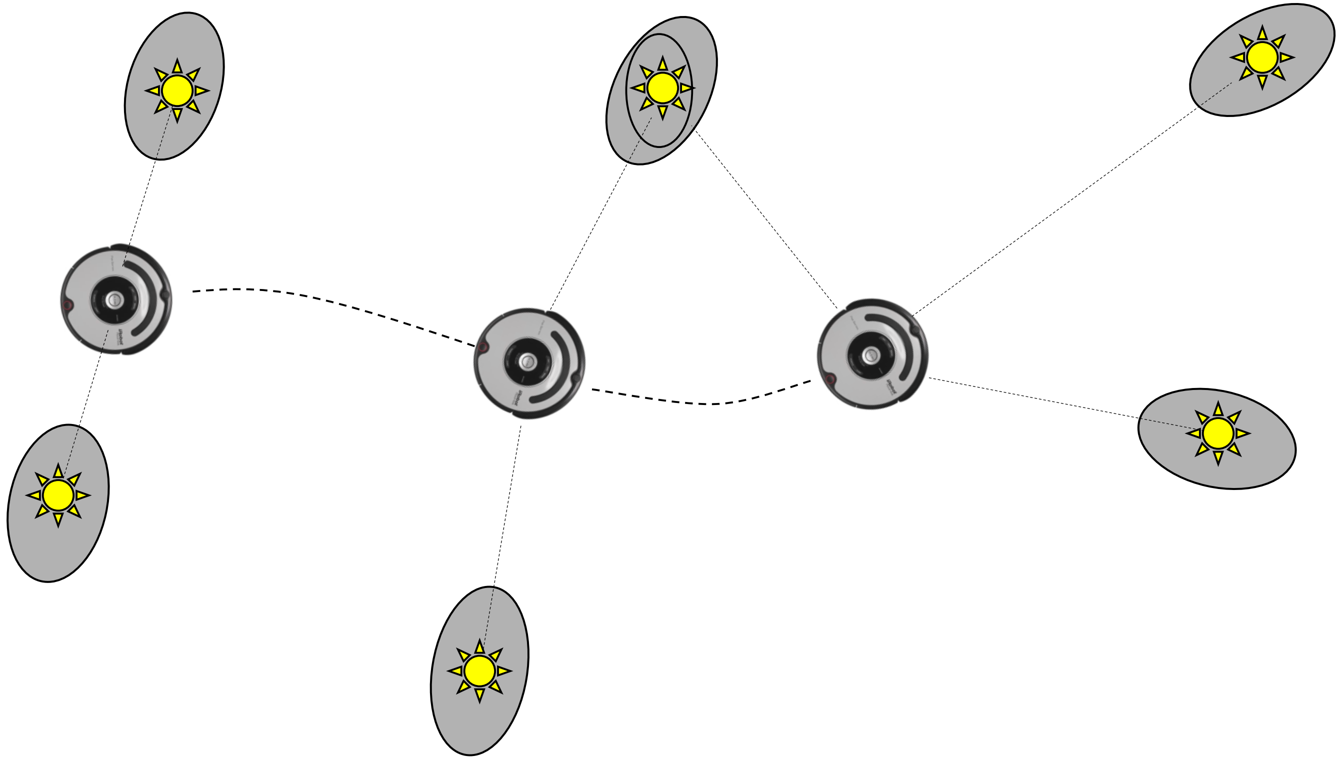


[Lu & Milios, 97; Gutmann, 98: Thrun 98; Burgard, 99; Konolige & al., 00; Thrun, 00; Arras, 99; Haehnel, 01;…]

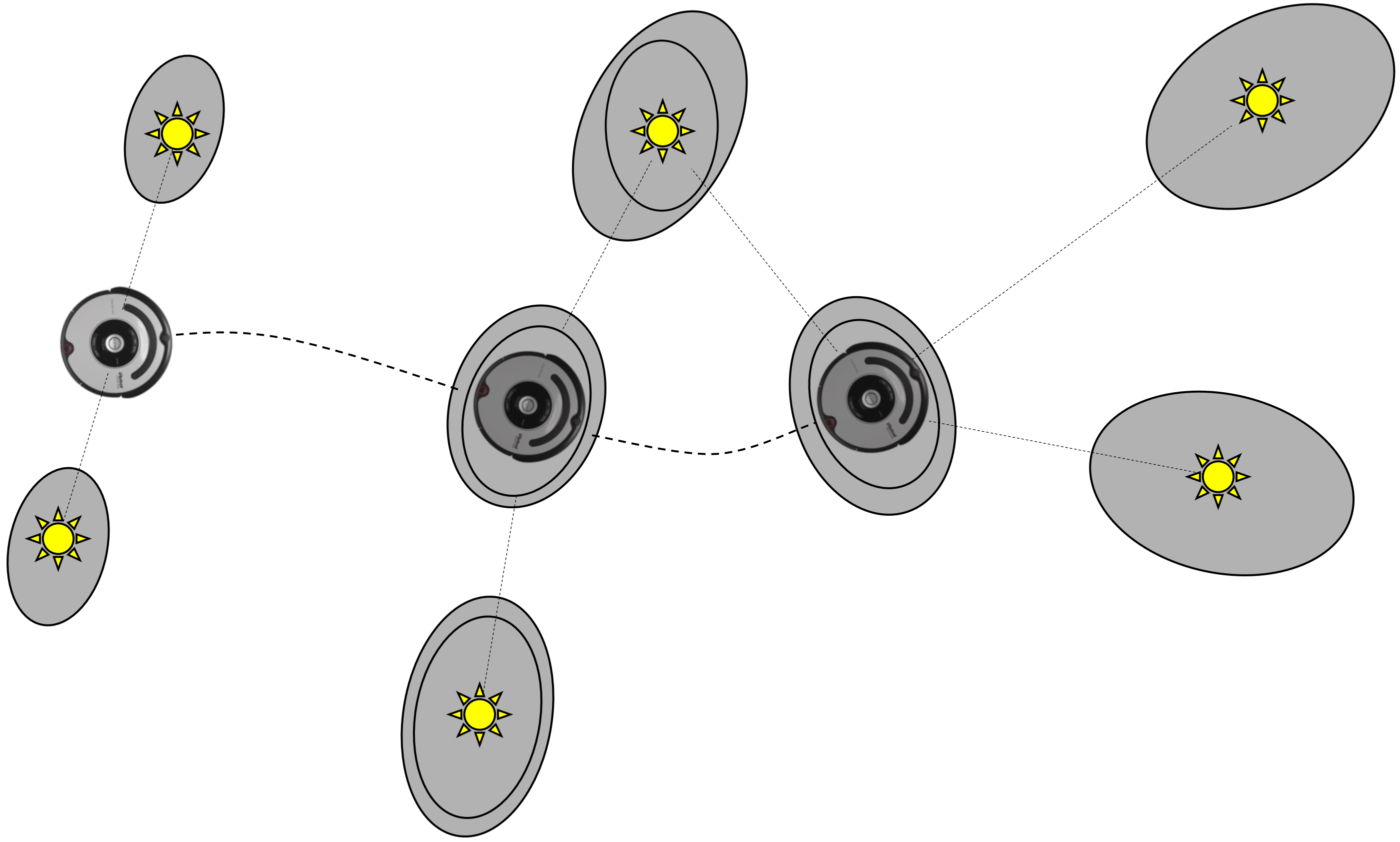
Localization … with known map



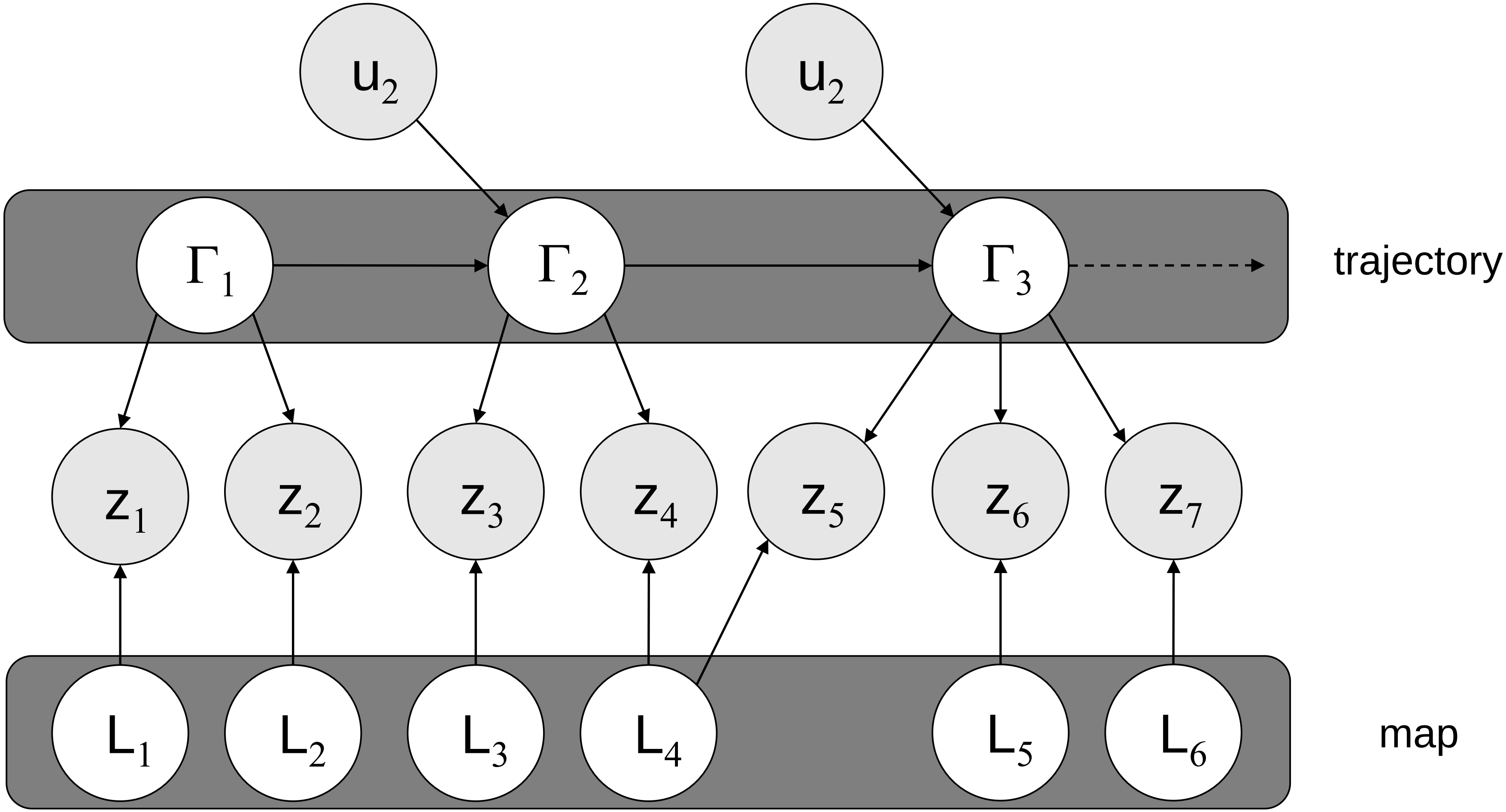
Mapping … with known poses



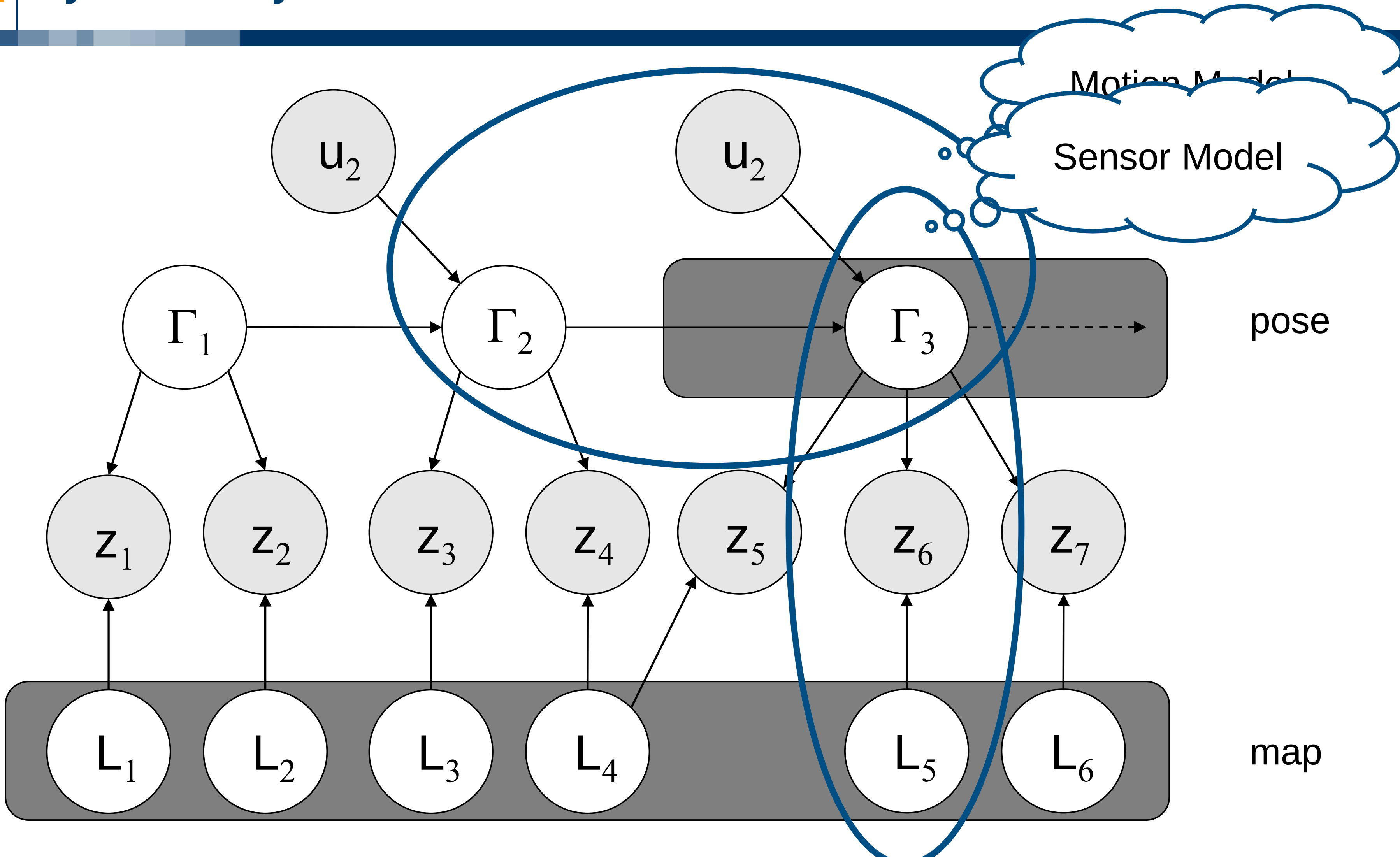
Simultaneous Localization and Mapping



Dynamic Bayes Network Inference and Full SLAM



$$\text{Smoothing} \quad : \quad p(\Gamma_{1:t}, l_1, \ldots, l_N \mid Z_{1:t}, U_{1:t})$$



$$\text{Filtering} \quad : \quad p(\Gamma_t, l_1, \ldots, l_N \mid Z_{1:t}, U_{1:t}) = \iiint_{1:t-1} p(\Gamma_{1:t}, l_1, \ldots, l_N \mid Z_{1:t}, U_{1:t})$$

Techniques for Generating Consistent Maps

Several techniques have been studied to obtain a consistent estimate of the joint probability of pose and map

- Scan matching
- EKF SLAM / UKF SLAM
- Fast-SLAM (Particle filter based)
- Probabilistic mapping with a single map and a posterior about poses (Mapping + Localization)
- Graph-SLAM, SEIFs
- ...

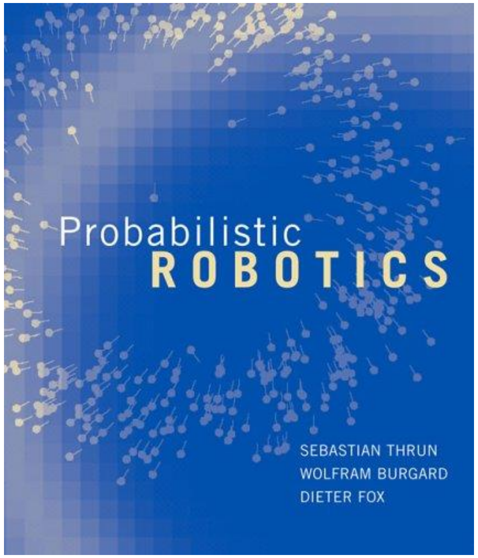
We won’t see the all of them! ☺

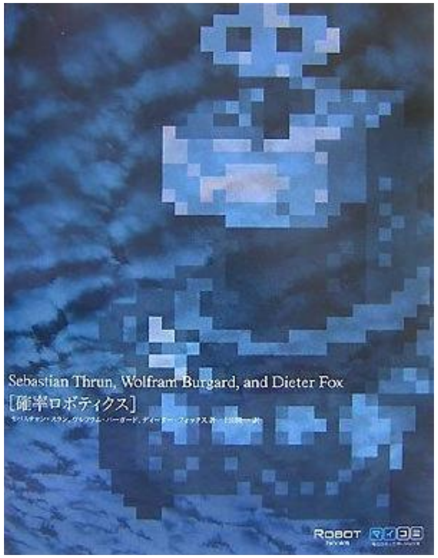
Let’s start with the basics! ;-)

Disclaimer …

These slides have been heavily "inspired" by the teaching material kindly provided with the book:

- **Probabilistic Robotics** by Sebastian Thrun, Dieter Fox, and Wolfram Burgard, MIT Press, 2005





Please refer to the original source for a deeper analysis and further references on the topic …

Bayes Filters: Framework

Given:

- Stream of observations *z* and action data *u:*

$$d_t = \{u_1, z_1 \ldots, u_t, z_t\}$$

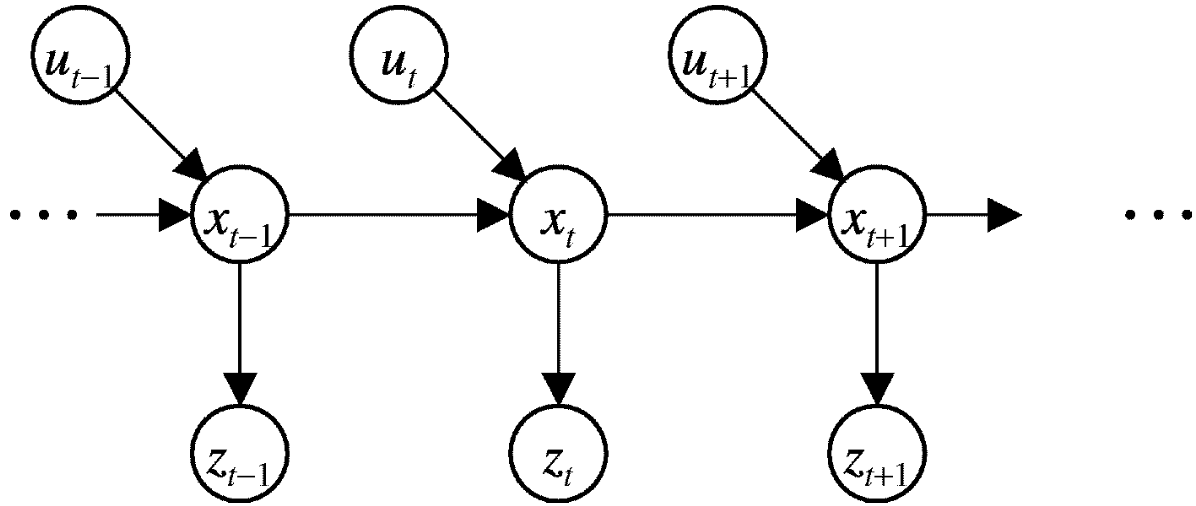
- Sensor model *P(z|x).*
- Action model *P(x|u,x')*.
- Prior probability of the system state *P(x).*

We want to compute:

- Estimate of the state *X* of a dynamical system.
- The posterior of the state is also called **Belief**:

$$Bel(x_t) = P(x_t \mid u_1, z_1 \ldots, u_t, z_t)$$

Markov Assumption



$$p(z_t \mid x_{0:t}, z_{1:t}, u_{1:t}) = p(z_t \mid x_t)$$

$$p(x_t \mid x_{1:t-1}, z_{1:t}, u_{1:t}) = p(x_t \mid x_{t-1}, u_t)$$

Underlying Assumptions

- Static world
- Independent noise
- Perfect model, no approximation errors

Bayes Filters

z = observation
u = action
x = state

$$\boxed{Bel(x_t)} = P(x_t \mid u_1, z_1 \ldots, u_t, z_t)$$

Bayes $$= \eta \; P(z_t \mid x_t, u_1, z_1, \ldots, u_t) \; P(x_t \mid u_1, z_1, \ldots, u_t)$$

Markov $$= \eta \; P(z_t \mid x_t) \; P(x_t \mid u_1, z_1, \ldots, u_t)$$

Total prob. $$= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_1, z_1, \ldots, u_t, x_{t-1}) \; P(x_{t-1} \mid u_1, z_1, \ldots, u_t) \; dx_{t-1}$$

Markov $$= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; P(x_{t-1} \mid u_1, z_1, \ldots, u_t) \; dx_{t-1}$$

Markov $$= \eta P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) P(x_{t-1} \mid u_1, z_1, \ldots, z_{t-1}) \; dx_{t-1}$$

$$\boxed{= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}}$$

Bayes Filter: The Algorithm

$$Bel(x_t) = \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}$$

How to represent such belief?

Algorithm Bayes_filter(*Bel(x), d*):

η=0

If *d* is a perceptual data item *z* then

For all *x* do

$$Bel'(x) = P(z \mid x) Bel(x)$$
$$\eta = \eta + Bel'(x)$$

For all *x* do

$$Bel'(x) = \eta^{-1} Bel'(x)$$

Else if *d* is an action data item *u* then

For all *x* do

$$Bel'(x) = \int P(x \mid u, x') \; Bel(x') \; dx'$$

Return *Bel'(x)*

Bayes Filters are Familiar!

$$Bel(x_t) = \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}$$

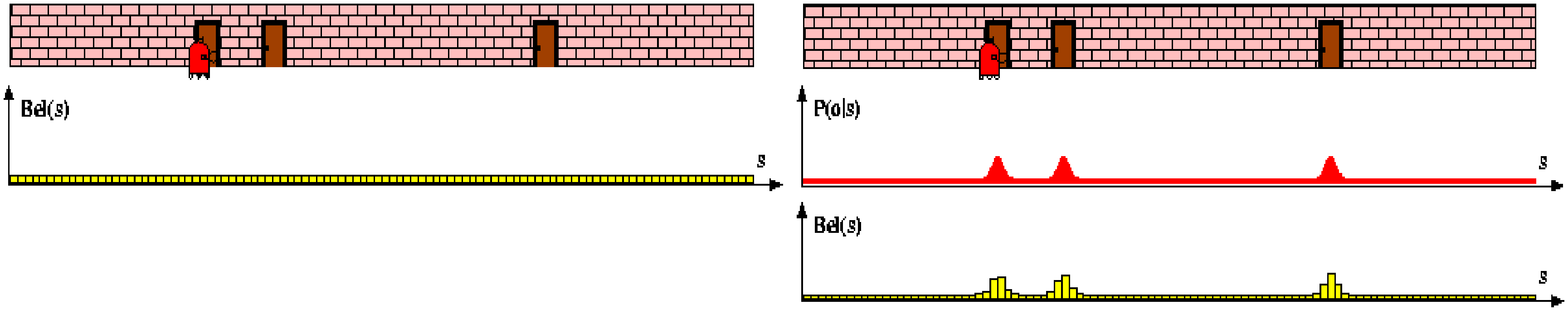
You might have met this filter already if you had something to do with:

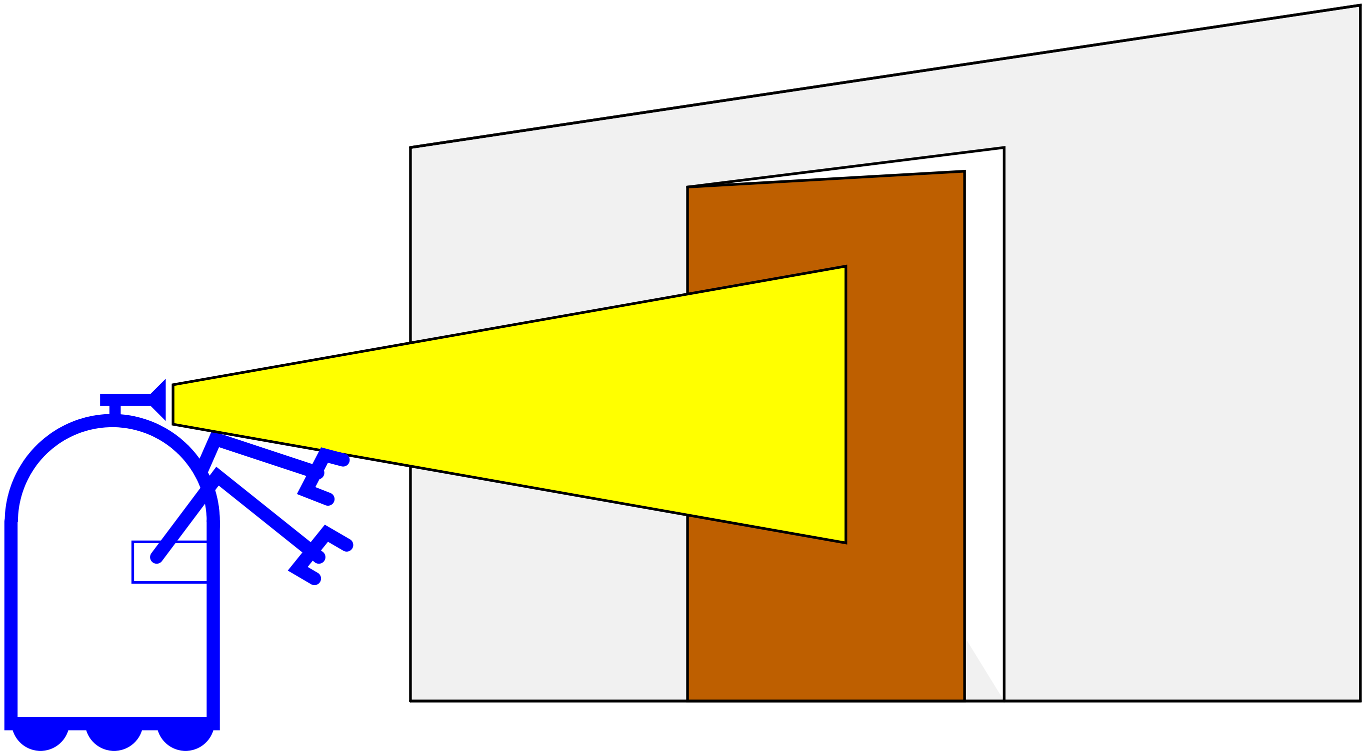
- Kalman filters
- Particle filters
- Hidden Markov models
- Dynamic Bayesian networks
- Partially Observable Markov Decision Processes (POMDPs)

Let have a closer look at:

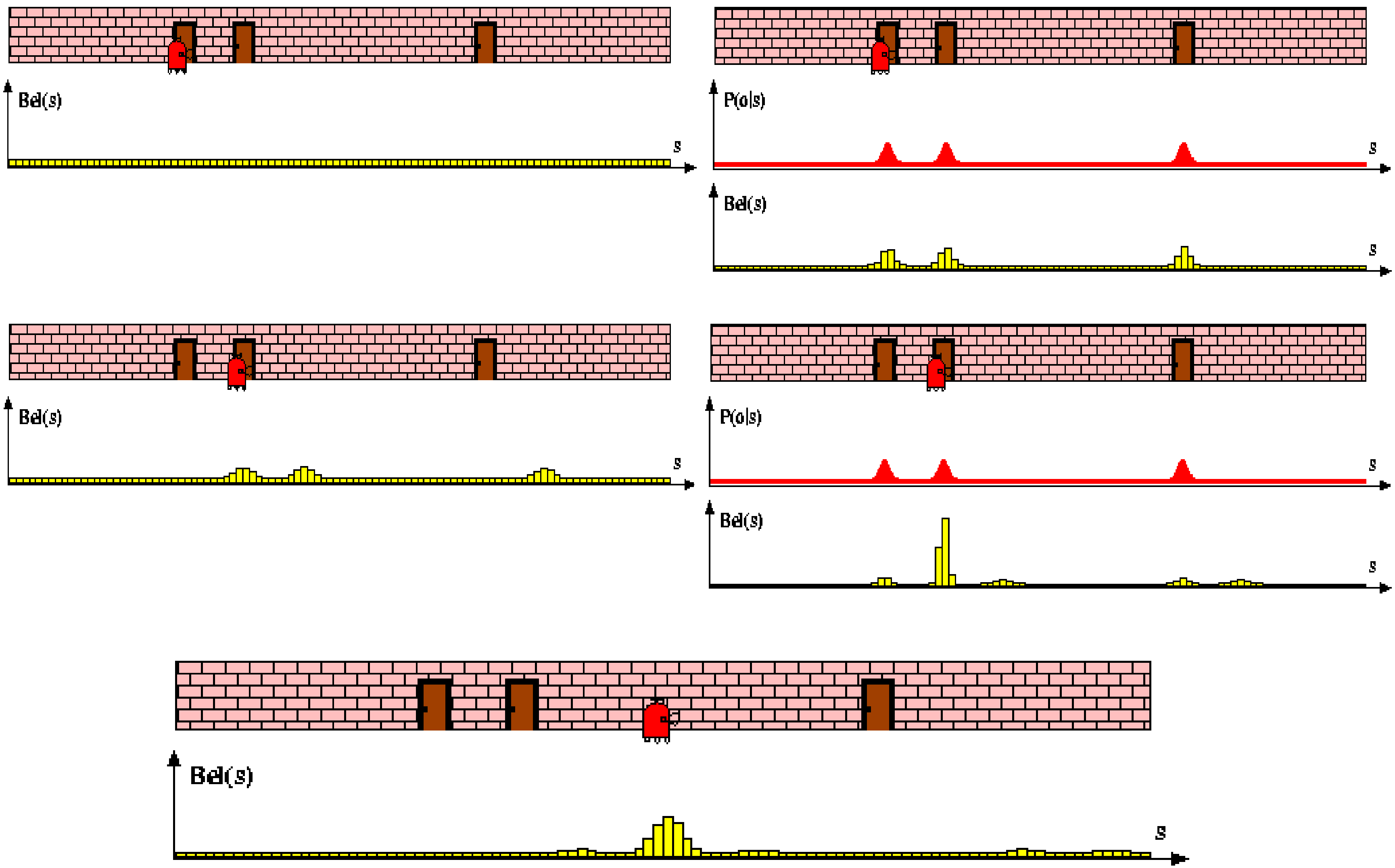
- Discrete filters
- Kalman filters
- Particle filters

Piecewise Constant Approximation





Piecewise Constant Approximation



Discrete Bayes Filter Algorithm

Algorithm Discrete_Bayes_filter(*Bel(x),d*):

h=0

If *d* is a perceptual data item *z* then

For all *x* do

$$Bel'(x) = P(z \mid x) Bel(x)$$

$$\eta = \eta + Bel'(x)$$

For all *x* do

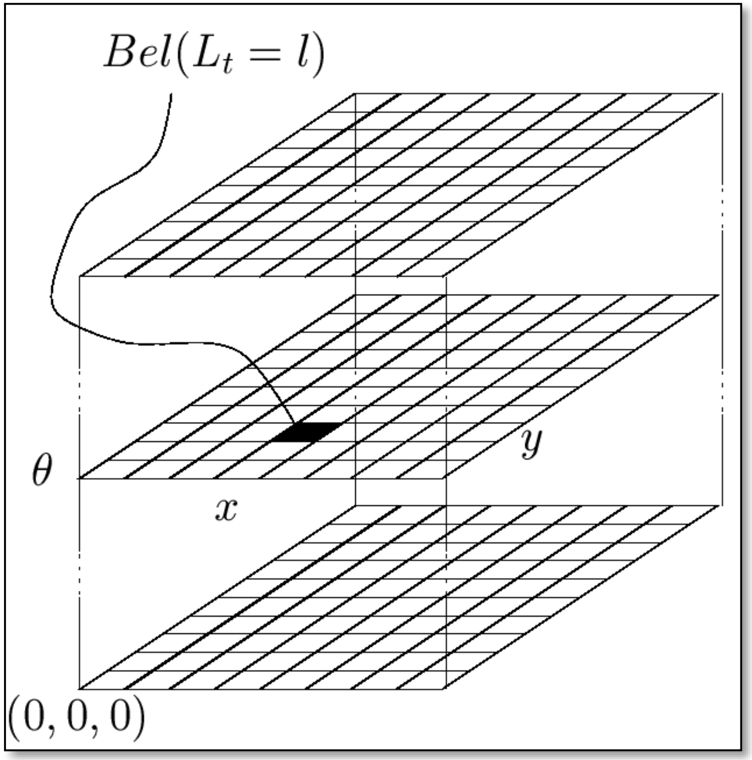
$$Bel'(x) = \eta^{-1} Bel'(x)$$

Else if *d* is an action data item *u* then

For all *x* do

$$Bel'(x) = \sum_{x'} P(x \mid u, x')\, Bel(x')$$

Return *Bel'(x)*



Implementation Tricks

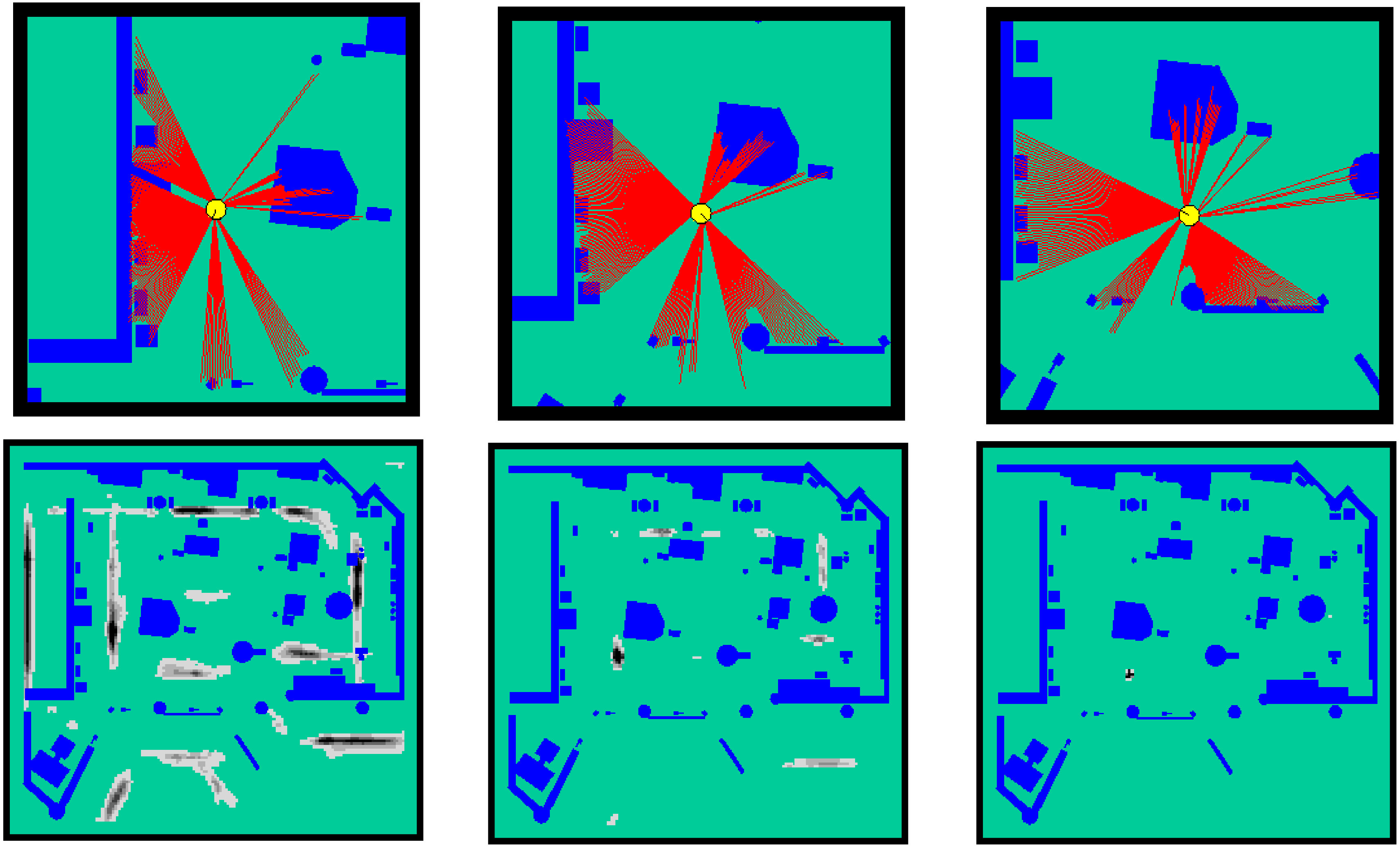
Belief update upon sensory input and normalization iterates over all cells

- When the belief is peaked (e.g., during position tracking), avoid updating irrelevant parts.
- Do not update entire sub-spaces of the state space and monitor whether the robot is de-localized or not by considering likelihood of observations given the active components

To update the belief upon robot motions, assumes a bounded Gaussian model; reduces the update from $O(n^2)$ to $O(n)$.

The update by shifting the data in the grid according to measured motion, then convolve the grid using a Gaussian Kernel.

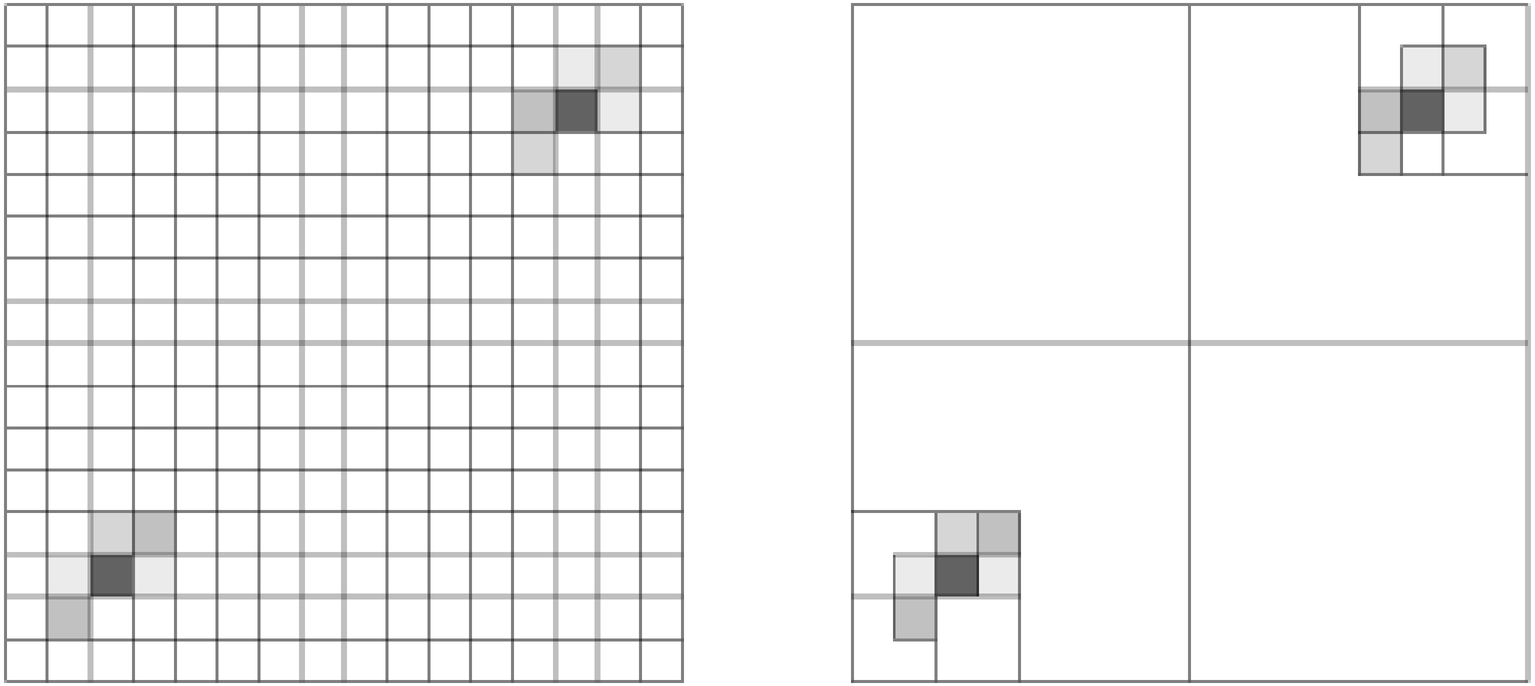
Grid-based Localization



Tree-based Representation

Filter complexity is exponential in the number of degrees of freedom, it can be sped up by representing density using a variant of octrees

- Efficient in space and time
- Multi-resolution



Bayes Filters are Familiar!

$$Bel(x_t) = \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}$$

You might have met this filter already if you had something to do with:

- Kalman filters
- Particle filters
- Hidden Markov models
- Dynamic Bayesian networks
- Partially Observable Markov Decision Processes (POMDPs)

Let have a closer look at:

- Discrete filters
- Kalman filters
- Particle filters

Bayes Filter Reminder

Prediction

$$\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1})\, bel(x_{t-1})\, dx_{t-1}$$

Correction

$$bel(x_t) = \eta\, p(z_t \mid x_t)\, \overline{bel}(x_t)$$

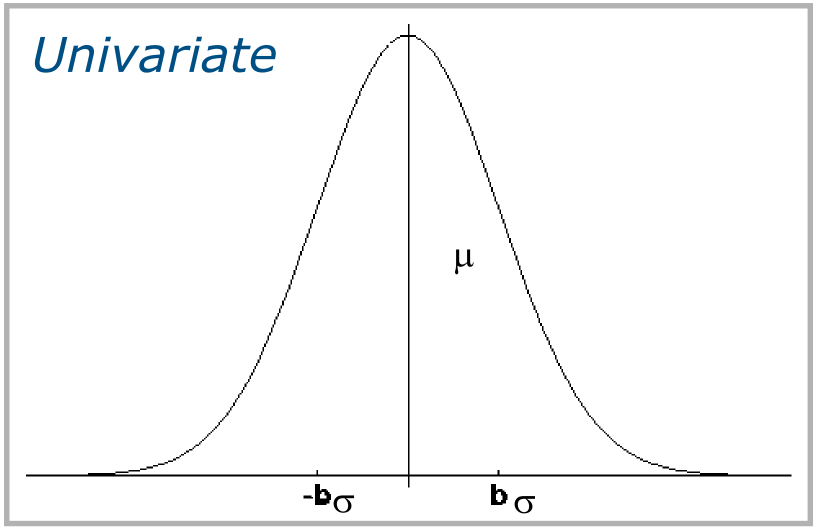
Can we easily compute these integrals (remind η is an integral too) in closed form for continuos distributions?

NO!

Is there any continuous distribution for which this is possible?

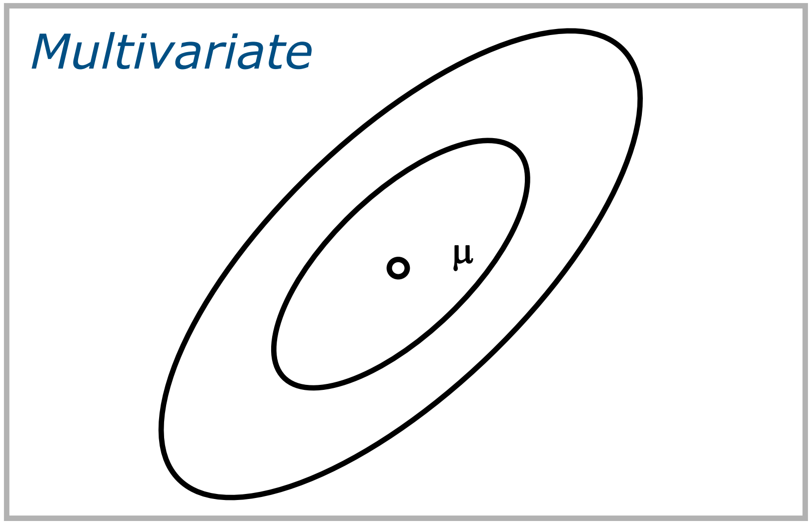
YES!

Gaussians



$$p(x) \sim N(\mu, \sigma^2):$$

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$



$$p(\mathbf{x}) \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}):$$

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{d/2}|\boldsymbol{\Sigma}|^{1/2}} e^{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^t \boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})}$$

Properties of Gaussians

Univariate

$$\left.\begin{array}{l} X \sim N(\mu,\sigma^2) \\ Y = aX + b \end{array}\right\} \Rightarrow \quad Y \sim N(a\mu + b, a^2\sigma^2)$$

$$\left.\begin{array}{l} X_1 \sim N(\mu_1,\sigma_1^{\,2}) \\ X_2 \sim N(\mu_2,\sigma_2^{\,2}) \end{array}\right\} \Rightarrow p(X_1)\cdot p(X_2) \sim N\left(\frac{\sigma_2^{\,2}}{\sigma_1^{\,2}+\sigma_2^{\,2}}\mu_1 + \frac{\sigma_1^{\,2}}{\sigma_1^{\,2}+\sigma_2^{\,2}}\mu_2, \quad \frac{1}{\sigma_1^{\,-2}+\sigma_2^{\,-2}}\right)$$

Multivariate

$$\left.\begin{array}{l} X \sim N(\mu,\Sigma) \\ Y = AX + B \end{array}\right\} \Rightarrow \quad Y \sim N(A\mu + B, A\Sigma A^T)$$

$$\left.\begin{array}{l} X_1 \sim N(\mu_1,\Sigma_1) \\ X_2 \sim N(\mu_2,\Sigma_2) \end{array}\right\} \Rightarrow p(X_1)\cdot p(X_2) \sim N\left(\frac{\Sigma_2}{\Sigma_1+\Sigma_2}\mu_1 + \frac{\Sigma_1}{\Sigma_1+\Sigma_2}\mu_2, \quad \frac{1}{\Sigma_1^{-1}+\Sigma_2^{-1}}\right)$$

Discrete Time Kalman Filter

Estimates the state *x* of a discrete-time controlled process that is governed by the linear stochastic difference equation

$$x_t = A_t x_{t-1} + B_t u_t + \varepsilon_t$$

with a measurement

$$z_t = C_t x_t + \delta_t$$

- A_t (n x n) describes state evolves from t to t-1 w/o controls or noise
- B_t (n x l) describes how control u_t changes the state from t to t-1
- C_t (k x n) that describes how to map the state x_t to an observation z_t
- ε_t δ_t random variables representing process and measurement noise assumed to be independent and normally distributed with covariance R_t and Q_t respectively.

Linear Gaussian Systems

Initial belief is normally distributed: $bel(x_0) = N\left(x_0; \mu_0, \Sigma_0\right)$

Dynamics are linear function of state and control plus additive noise:

$$x_t = A_t x_{t-1} + B_t u_t + \varepsilon_t$$

$$p(x_t \mid u_t, x_{t-1}) = N\left(x_t; A_t x_{t-1} + B_t u_t, R_t\right)$$

Linear Gaussian Systems: Dynamics

$$\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1}) \cdot bel(x_{t-1})\, dx_{t-1}$$

$$\sim N\left(x_t; A_t x_{t-1} + B_t u_t, R_t\right) \quad \sim N\left(x_{t-1}; \mu_{t-1}, \Sigma_{t-1}\right)$$

$$\overline{bel}(x_t) = \eta \int \exp\left\{-\frac{1}{2}(x_t - A_t x_{t-1} - B_t u_t)^T R_t^{-1} (x_t - A_t x_{t-1} - B_t u_t)\right\}$$

$$\exp\left\{-\frac{1}{2}(x_{t-1} - \mu_{t-1})^T \Sigma_{t-1}^{-1} (x_{t-1} - \mu_{t-1})\right\} dx_{t-1}$$

$$\overline{bel}(x_t) = \begin{cases} \bar{\mu}_t = A_t \mu_{t-1} + B_t u_t \\ \bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t \end{cases}$$

Linear Gaussian Systems: Observations

Observations are linear function of state plus additive noise:

$$z_t = C_t x_t + \delta_t$$

$$p(z_t \mid x_t) = N\left(z_t ; C_t x_t , Q_t\right)$$

$$\begin{array}{cccc} bel(x_t) = & \eta \quad p(z_t \mid x_t) & \cdot & \overline{bel}(x_t) \\ & \sim N\left(z_t ; C_t x_t , Q_t\right) & & \sim N\left(x_t ; \bar{\mu}_t , \bar{\Sigma}_t\right) \end{array}$$

$$bel(x_t) = \eta \exp\left\{-\frac{1}{2}(z_t - C_t x_t)^T Q_t^{-1}(z_t - C_t x_t)\right\}\exp\left\{-\frac{1}{2}(x_t - \bar{\mu}_t)^T \bar{\Sigma}_t^{-1}(x_t - \bar{\mu}_t)\right\}$$

$$bel(x_t) = \begin{cases} \mu_t = \bar{\mu}_t + K_t(z_t - C_t \bar{\mu}_t) \\ \Sigma_t = (I - K_t C_t)\bar{\Sigma}_t \end{cases} \quad \text{with} \quad K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$

Kalman Filter Algorithm

Algorithm Kalman_filter(μ_{t-1}, Σ_{t-1}, u_t, z_t):

Prediction:

$$\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$$
$$\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$$

Correction:

$$K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$
$$\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$$
$$\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$$

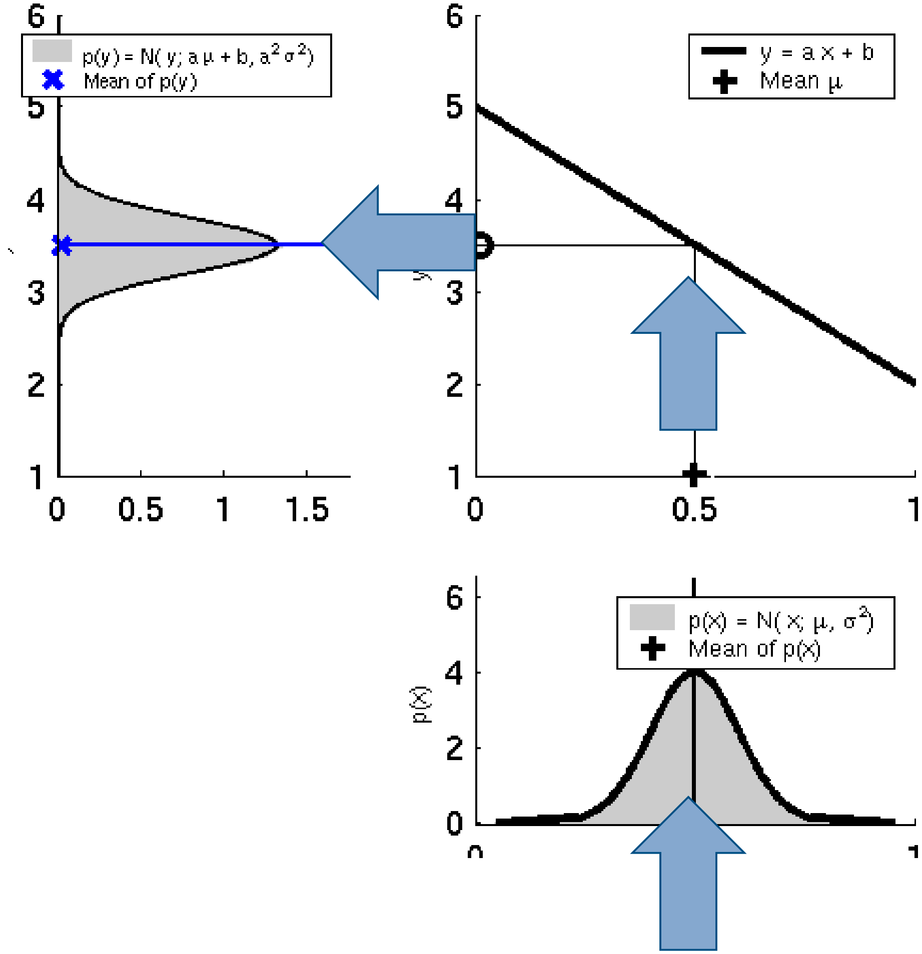
Return μ_t, Σ_t

- Highly efficient: Polynomial in measurement dimensionality *k* and state dimensionality *n*: $O(k^{2.376} + n^2)$
- Optimal for linear Gaussian systems ☺
- Most robotics systems are nonlinear ☹

Most realistic robotic problems involve nonlinear functions

$$x_t = A_t x_{t-1} + B_t u_t + \varepsilon_t$$

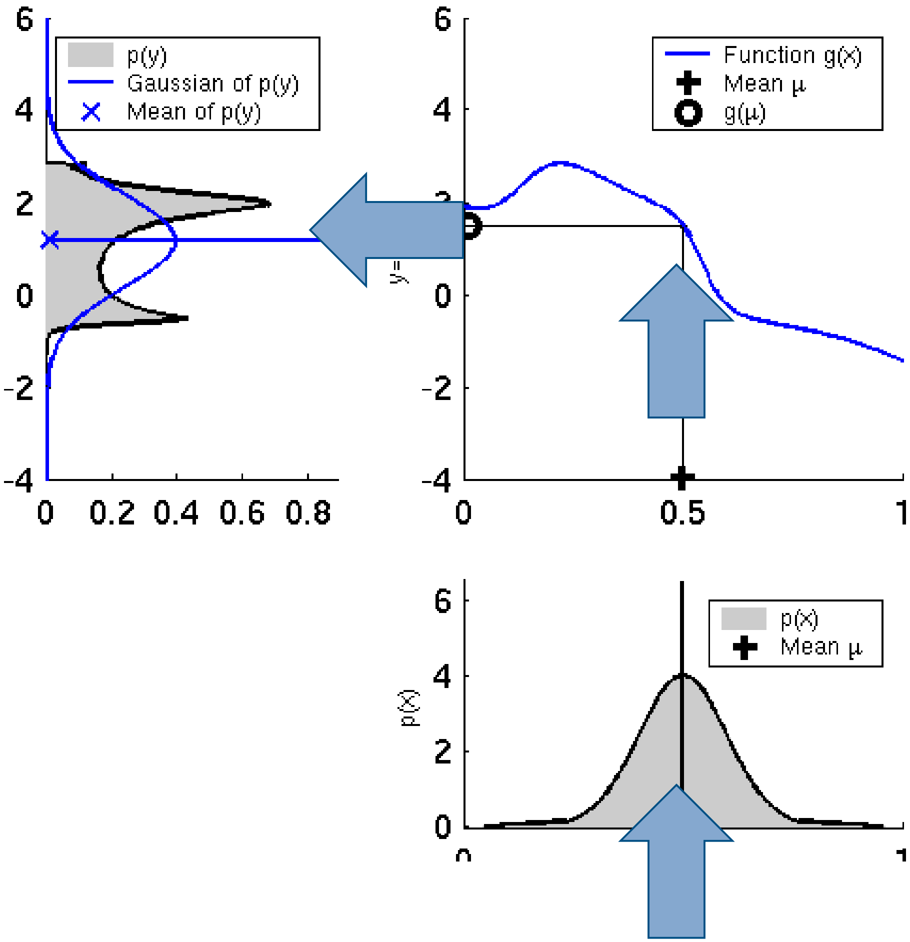
$$z_t = C_t x_t + \delta_t$$



Most realistic robotic problems involve nonlinear functions

$$x_t = g(u_t, x_{t-1})$$

$$z_t = h(x_t)$$



EKF: First Order Taylor Series Expansion

Prediction:

$$g(u_t, x_{t-1}) \approx g(u_t, \mu_{t-1}) + \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}} (x_{t-1} - \mu_{t-1})$$

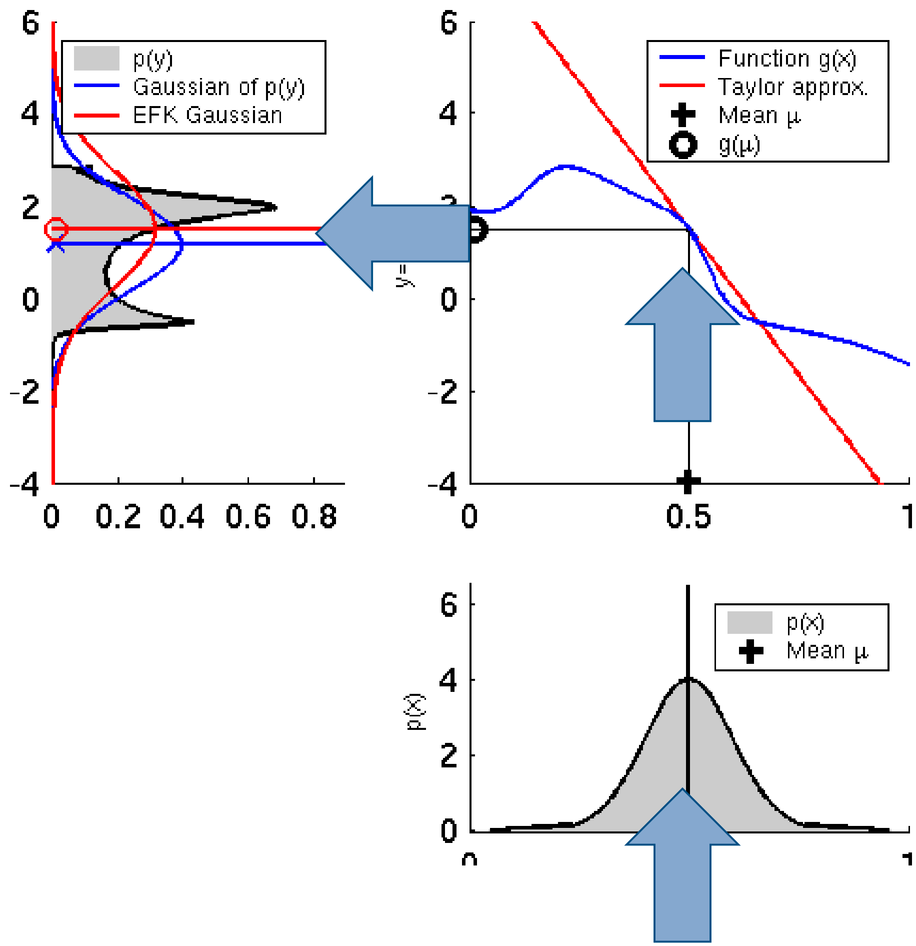
$$g(u_t, x_{t-1}) \approx g(u_t, \mu_{t-1}) + G_t \ (x_{t-1} - \mu_{t-1})$$

Correction:

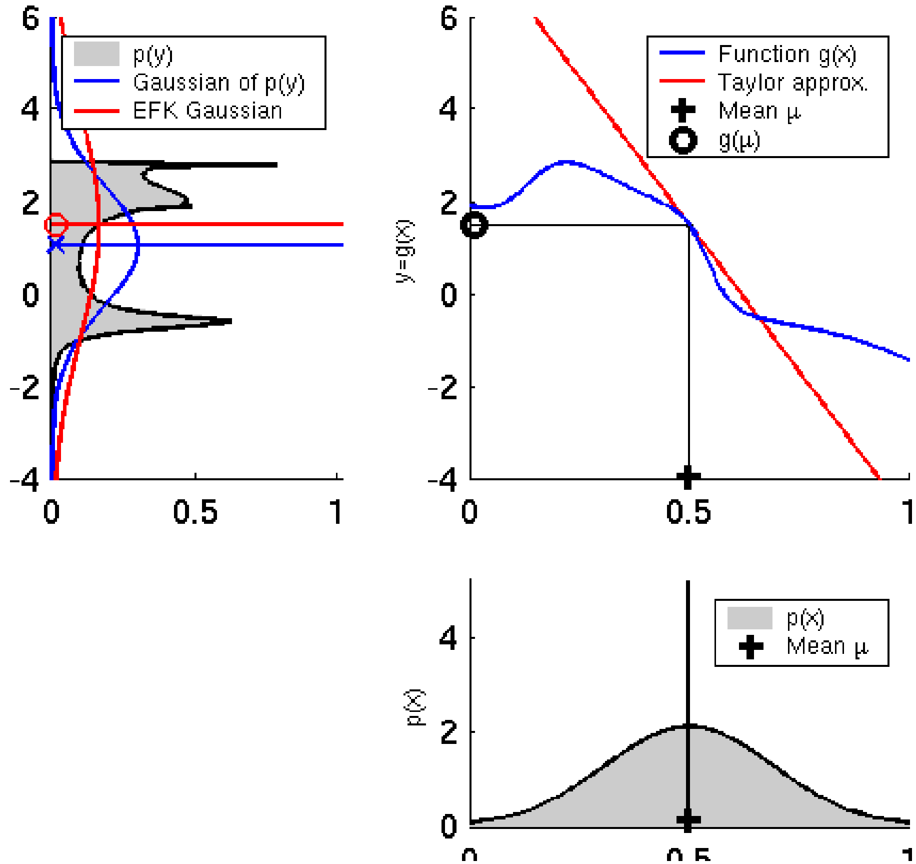
$$h(x_t) \approx h(\bar{\mu}_t) + \frac{\partial h(\bar{\mu}_t)}{\partial x_t} (x_t - \bar{\mu}_t)$$

$$h(x_t) \approx h(\bar{\mu}_t) + H_t \ (x_t - \bar{\mu}_t)$$

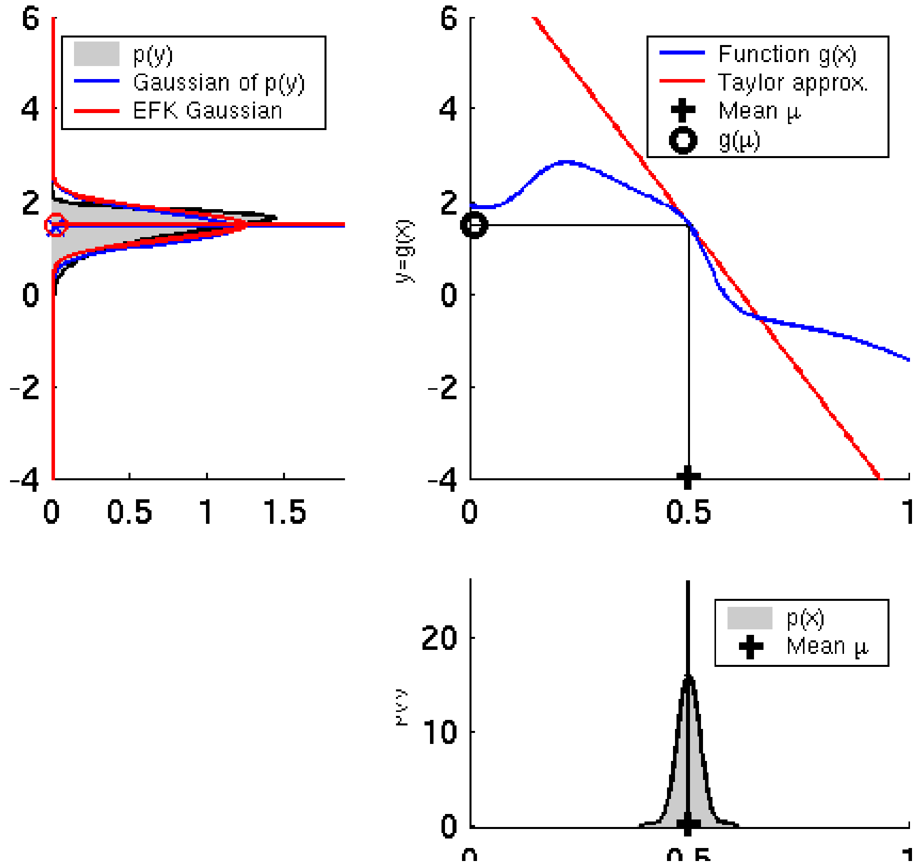
EKF Linearization (1)



EKF Linearization (2)



EKF Linearization (3)



EKF Algorithm

Extended_Kalman_filter(μ_{t-1}, Σ_{t-1}, u_t, z_t):

$$G_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}} \qquad H_t = \frac{\partial h(\bar{\mu}_t)}{\partial x_t}$$

Prediction:

$$\bar{\mu}_t = g(u_t, \mu_{t-1}) \longleftarrow \bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$$

$$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t \longleftarrow \bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$$

Correction:

$$K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1} \longleftarrow K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$

$$\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t)) \longleftarrow \mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$$

$$\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t \longleftarrow \Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$$

Return μ_t, Σ_t

Bayes Filters are Familiar!

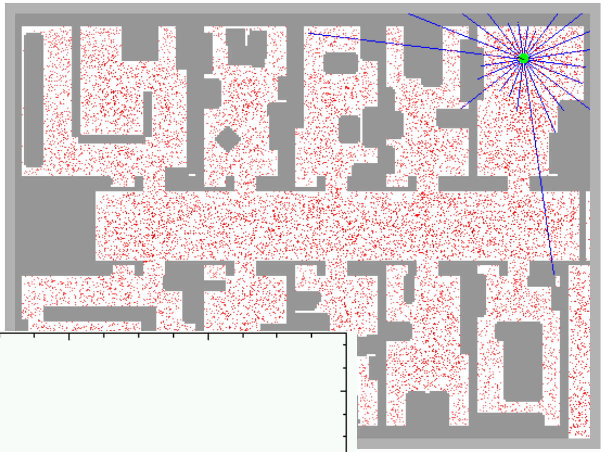
$$Bel(x_t) = \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}$$

You might have met this filter already if you had something to do with:

- Kalman filters
- Particle filters
- Hidden Markov models
- Dynamic Bayesian networks
- Partially Observable Markov Decision Processes (POMDPs)

Let have a closer look at:

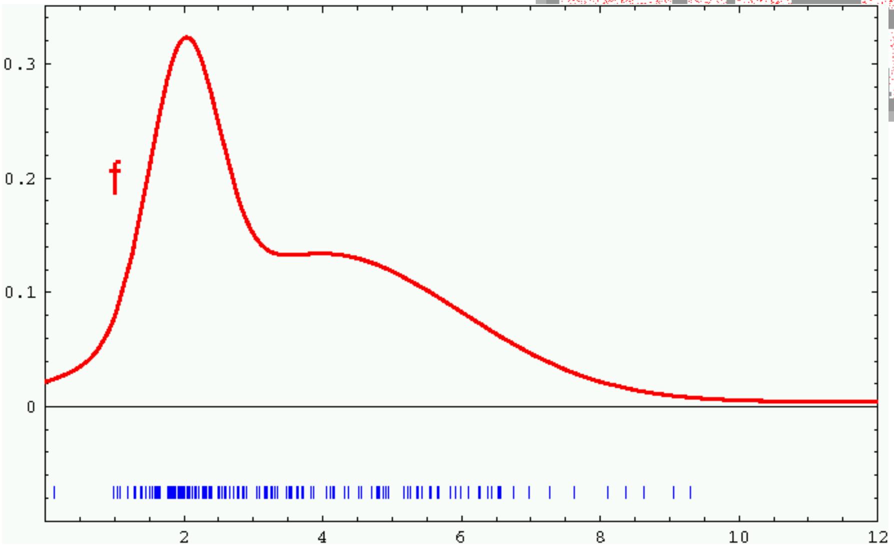
- Discrete filters
- Kalman filters
- Particle filters



Represent belief by random samples

Estimation of non-Gaussian, nonlinear processes

- Monte Carlo filter
- Survival of the fittest
- Condensation
- Bootstrap filter
- Particle filter
- …

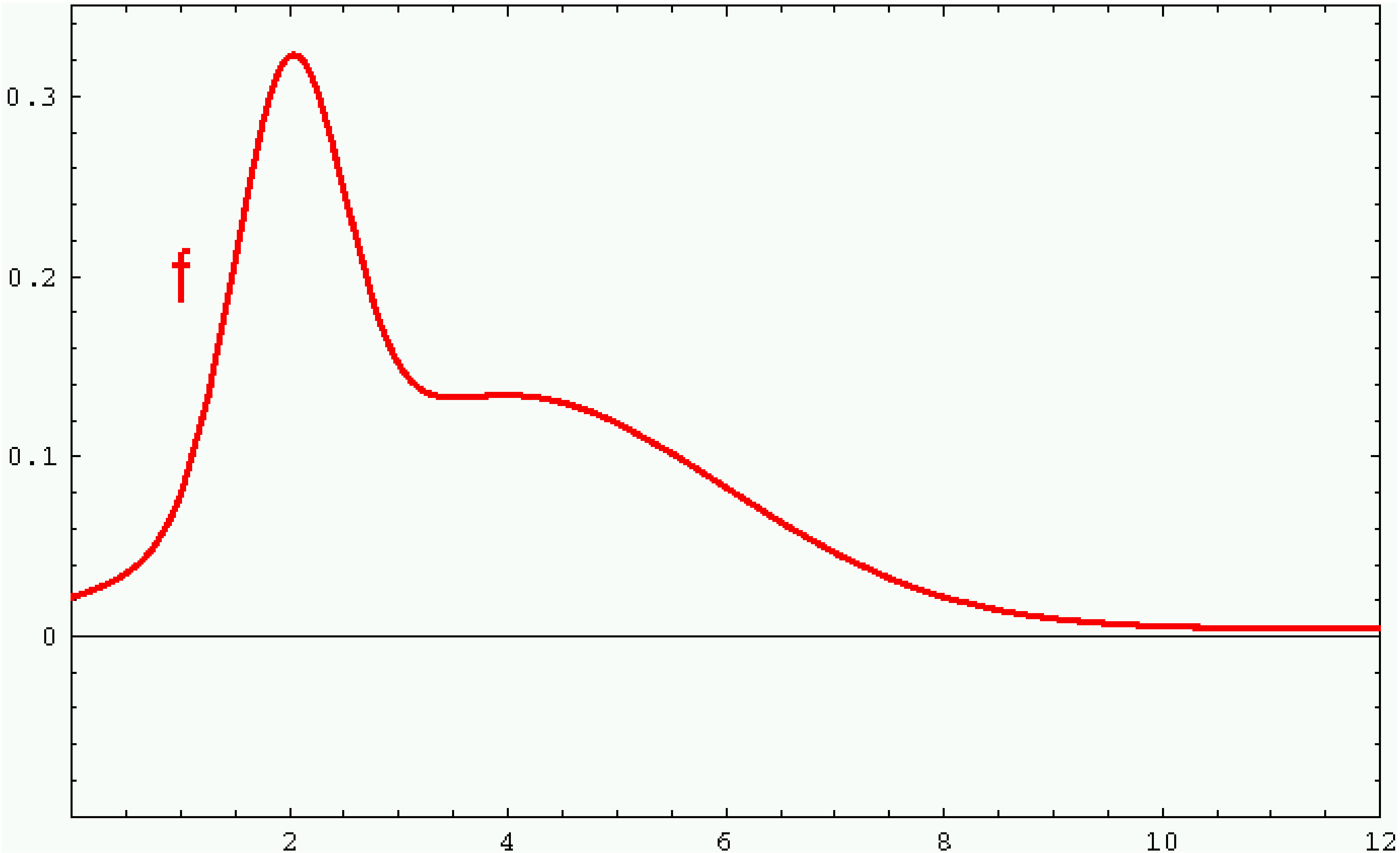


Filtering: [Rubin, 88], [Gordon et al., 93], [Kitagawa 96]

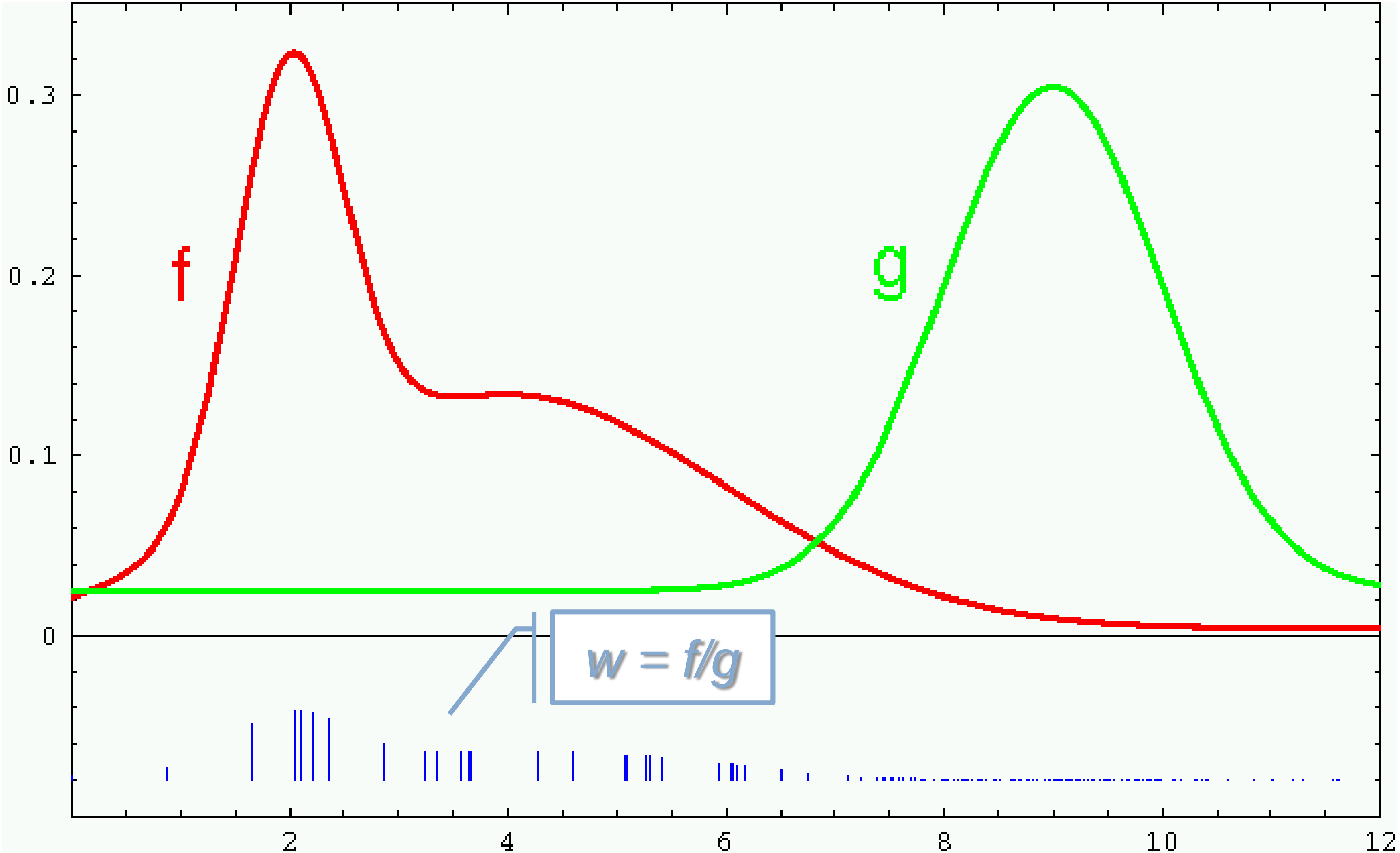
Computer vision: [Isard and Blake 96, 98]

Dynamic Bayesian Networks: [Kanazawa et al., 95]

Importance Resampling

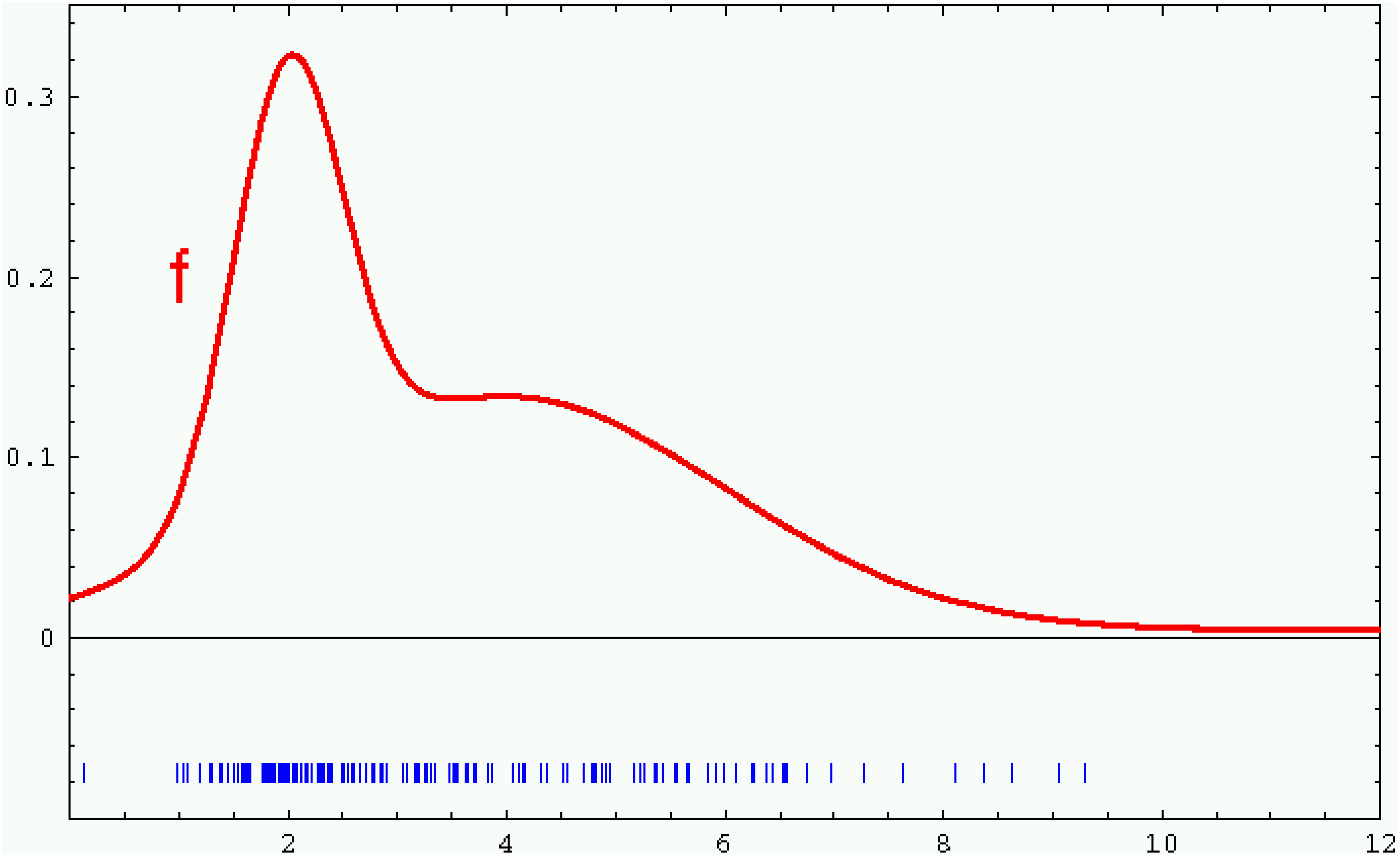


Importance Resampling



Matteo Matteucci – matteo.matteucci@polimi.it

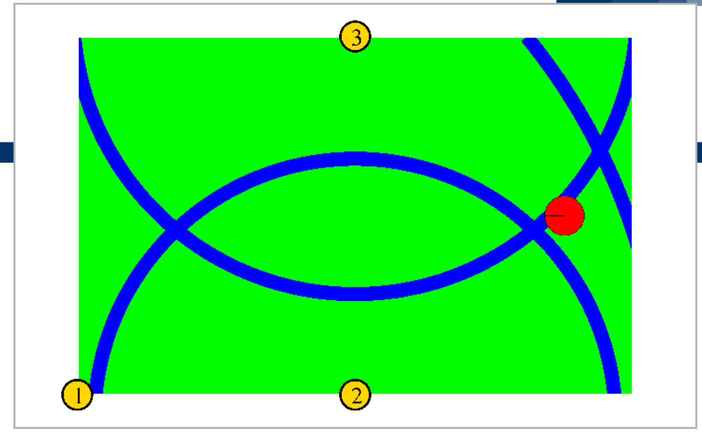
Importance Resampling (with smoothing)

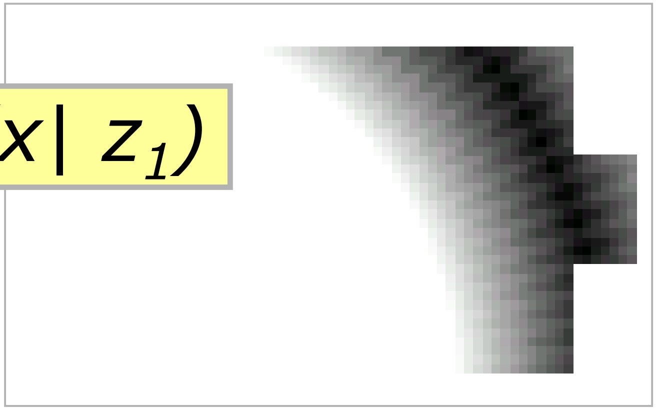


Matteo Matteucci – matteo.matteucci@polimi.it

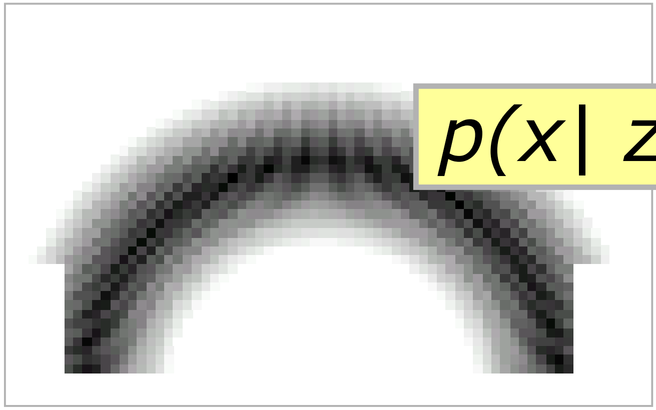
A Four Legged Example …

Involved Distributions

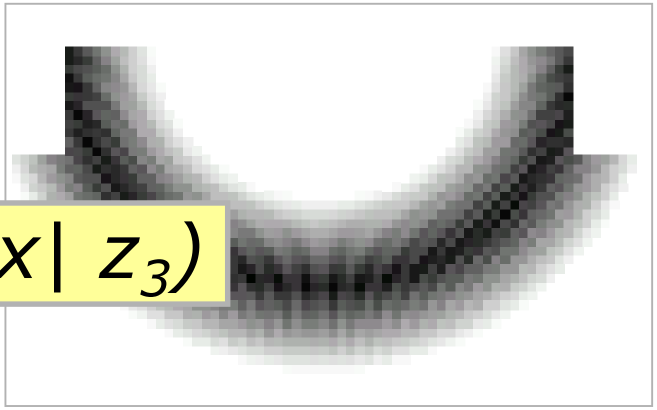




$p(x|\, z_1)$

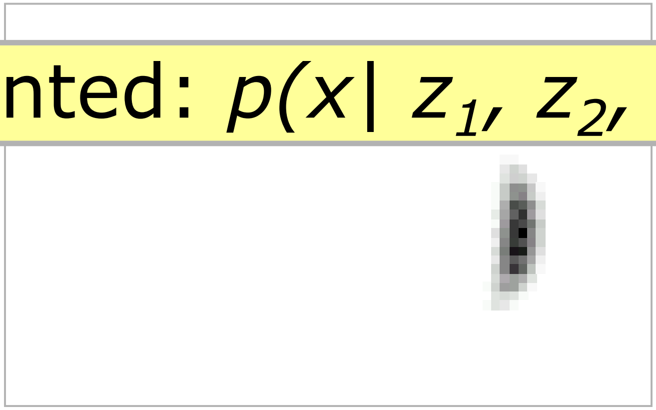


$p(x|\, z_2)$



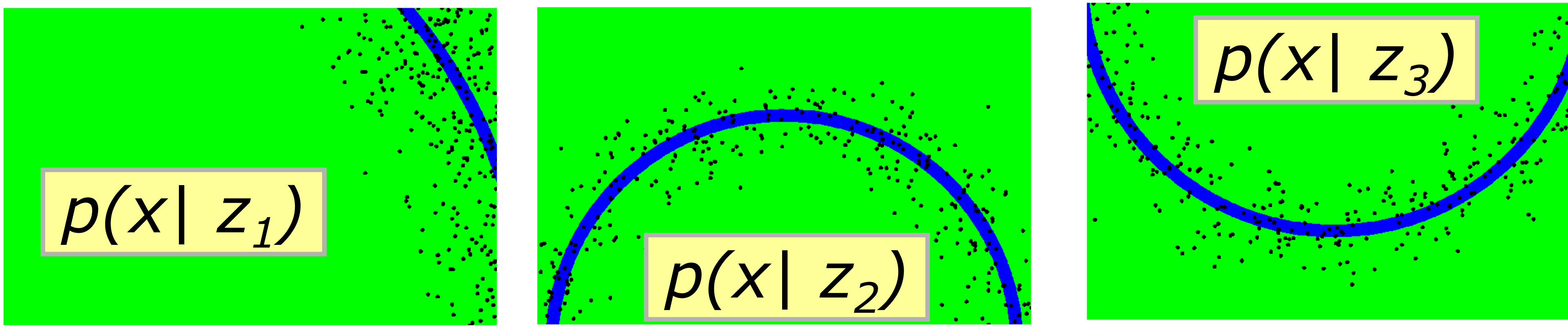
$p(x|\, z_3)$

Wanted: $p(x|\, z_1, z_2, z_3)$



This is (somehow) easy!

Draw samples from $p(x|z_i)$ using the detection parameters and some noise

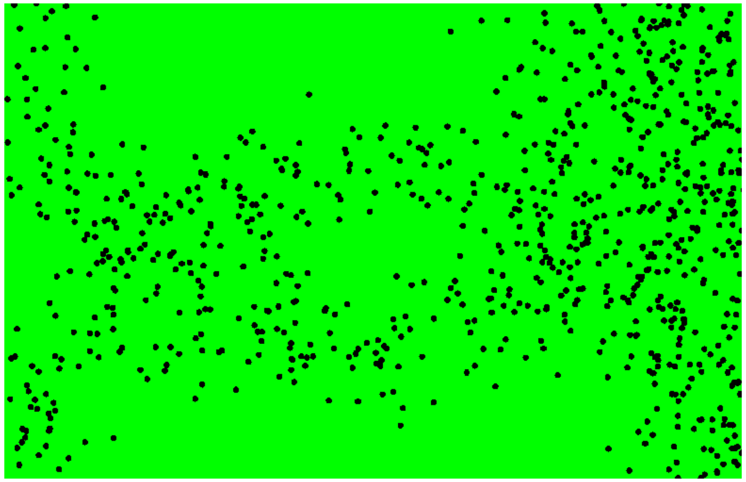


$$\text{Target distributi on f}: p(x \mid z_1, z_2, \ldots, z_n) = \frac{\prod_k p(z_k \mid x) \quad p(x)}{p(z_1, z_2, \ldots, z_n)}$$

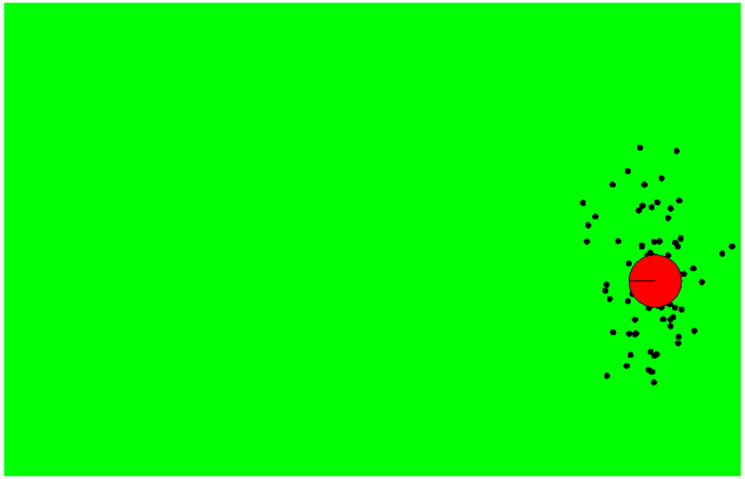
$$\text{Sampling distributi on g}: p(x \mid z_l) = \frac{p(z_l \mid x)\, p(x)}{p(z_l)}$$

$$\text{Importance weights w}: \frac{f}{g} = \frac{p(x \mid z_1, z_2, \ldots, z_n)}{p(x \mid z_l)} = \frac{p(z_l) \prod_{k \neq l} p(z_k \mid x)}{p(z_1, z_2, \ldots, z_n)}$$

Importance Sampling with Resampling

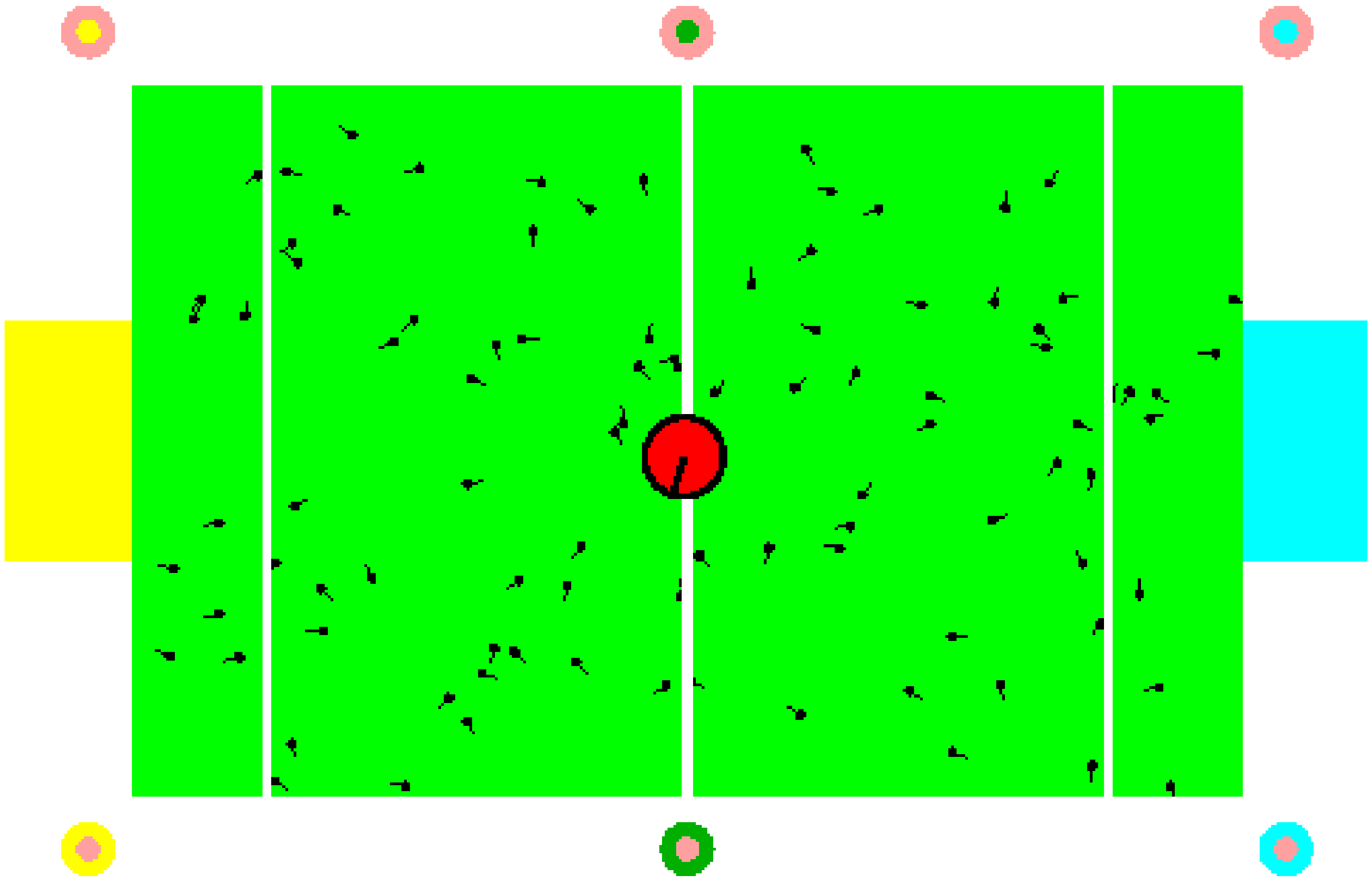


Weighted samples



After resampling

Localization for AIBO robots



Particle Filter Algorithm

$$\underbrace{Bel(x_t) = \eta\ p(z_t \mid x_t) \overbrace{\int p(x_t \mid x_{t-1}, u_{t-1})\ Bel(x_{t-1})\ dx_{t-1}}^{g}}_{f}$$

draw x^i_{t-1} from $Bel(x_{t-1})$

draw x^i_t from $p(x_t \mid x^i_{t-1}, u_{t-1})$

Importance factor for x^i_t:

$$w^i_t = \frac{\text{target distribution}}{\text{proposal distribution}} = \frac{\eta\ p(z_t \mid x_t)\ p(x_t \mid x_{t-1}, u_{t-1})\ Bel(x_{t-1})}{p(x_t \mid x_{t-1}, u_{t-1})\ Bel(x_{t-1})} \propto p(z_t \mid x_t)$$

Particle Filter Algorithm

Algorithm **particle_filter**(S_{t-1}, u_{t-1}, z_t):

$$S_t = \emptyset, \quad \eta = 0$$

For $i = 1 \ldots n$ ***Generate new samples***

Sample index *j(i)* from the discrete distribution given by w_{t-1}

Sample x_t^i from $p(x_t \mid x_{t-1}, u_{t-1})$ using $x_{t-1}^{j(i)}$ and u_{t-1}

$w_t^i = p(z_t \mid x_t^i)$ ***Compute importance weight***

$\eta = \eta + w_t^i$ ***Update normalization factor***

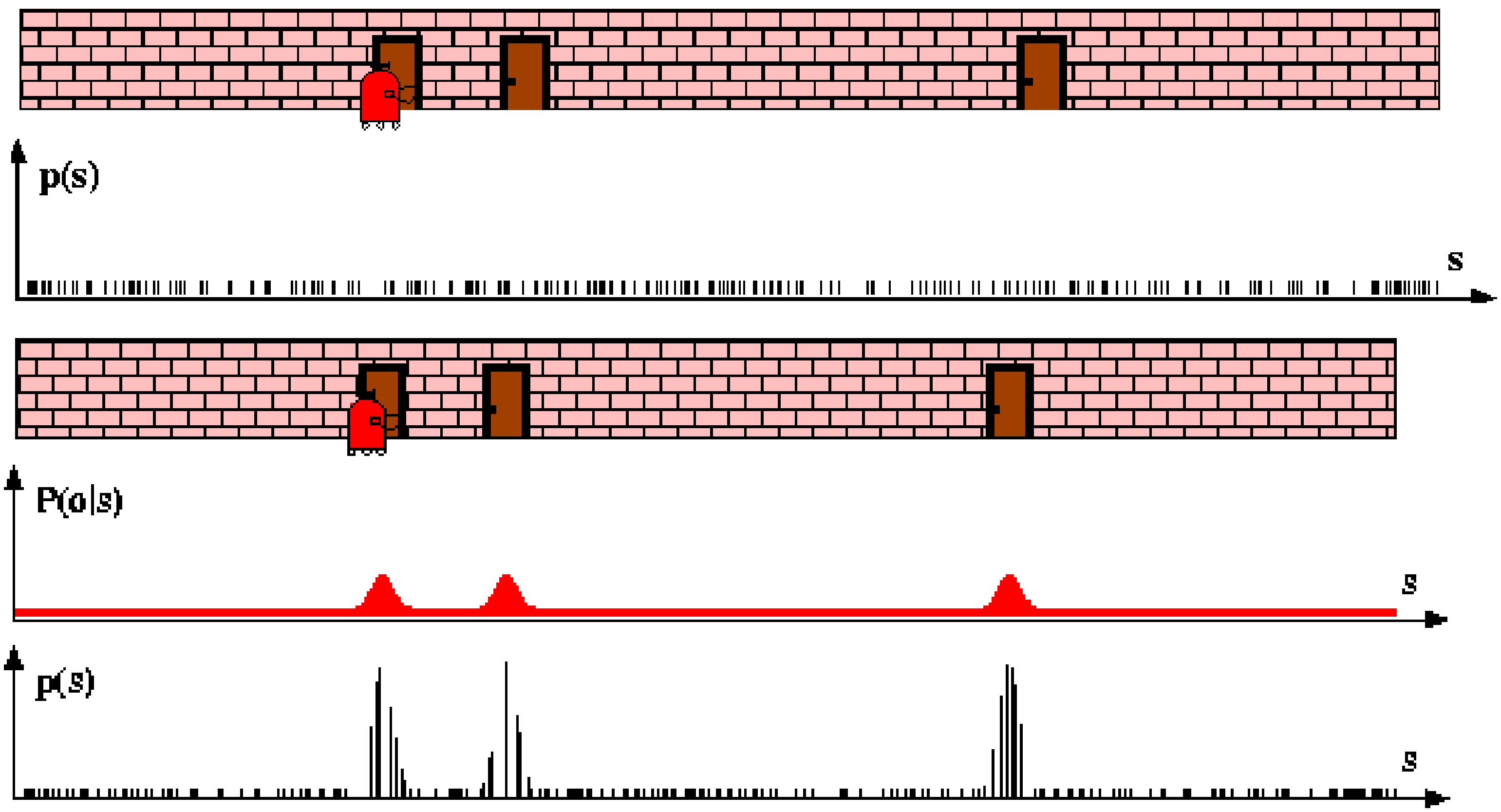
$S_t = S_t \cup \{< x_t^i, w_t^i >\}$ ***Insert***

For $i = 1 \ldots n$

$w_t^i = w_t^i / \eta$ ***Normalize weights***

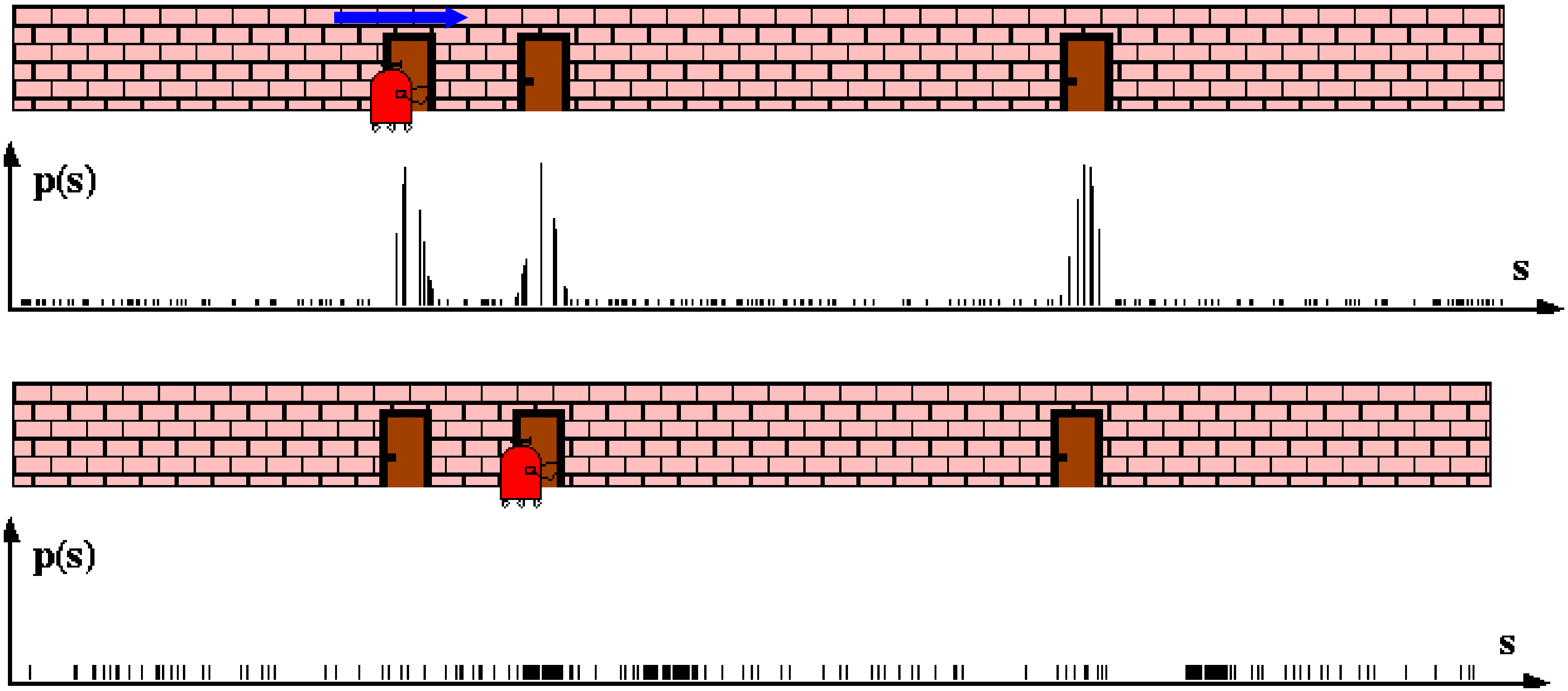
$$Bel(x) \leftarrow \alpha\ p(z|x)\, Bel^{-}(x)$$

$$w \leftarrow \frac{\alpha\ p(z|x)\, Bel^{-}(x)}{Bel^{-}(x)} = \alpha\ p(z|x)$$



Robot Motion

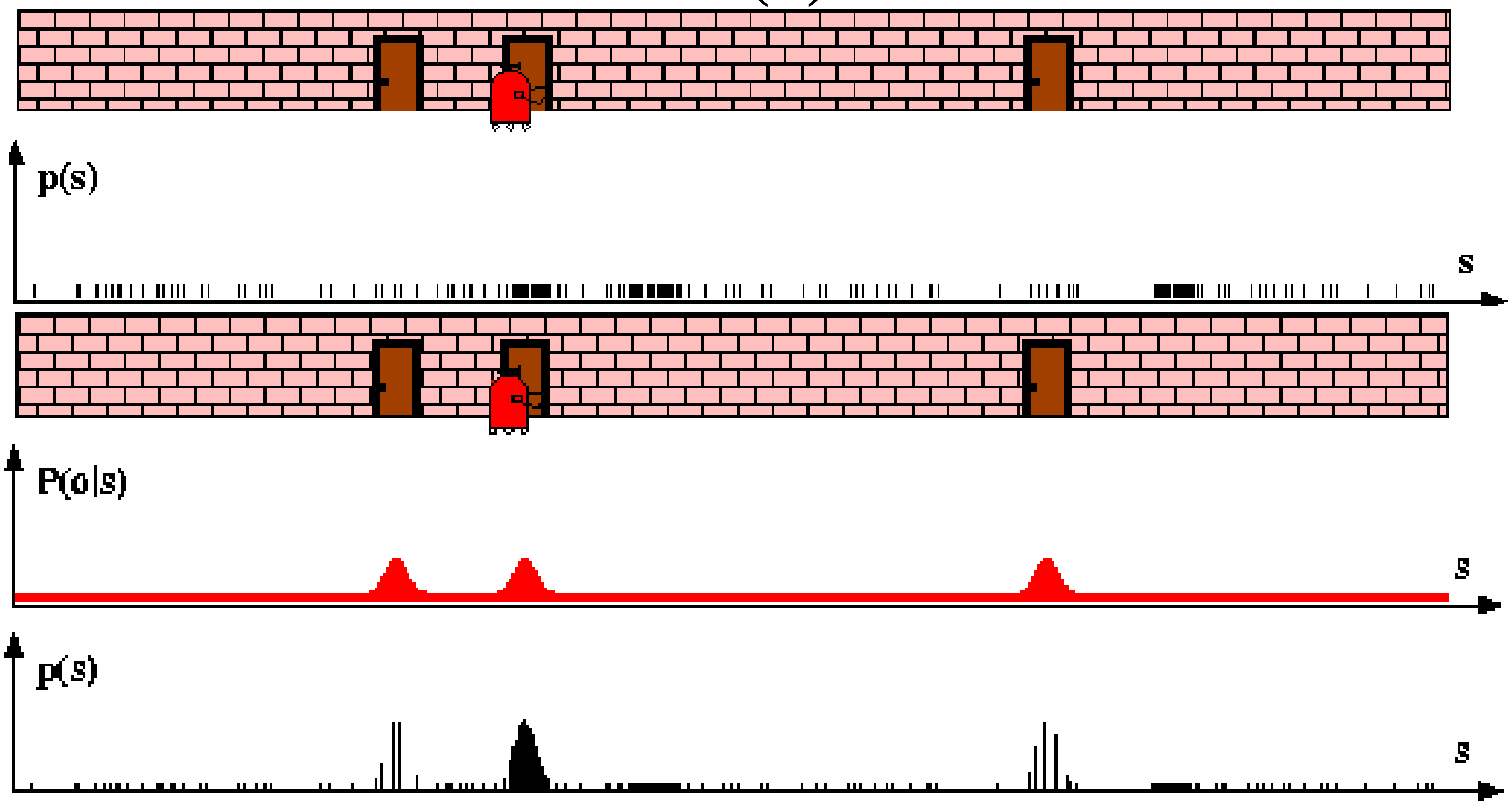
$$Bel^{-}(x) \leftarrow \int p(x \mid u, x')\, Bel(x')\ \mathrm{d}x'$$



Sensor Information: Importance Sampling

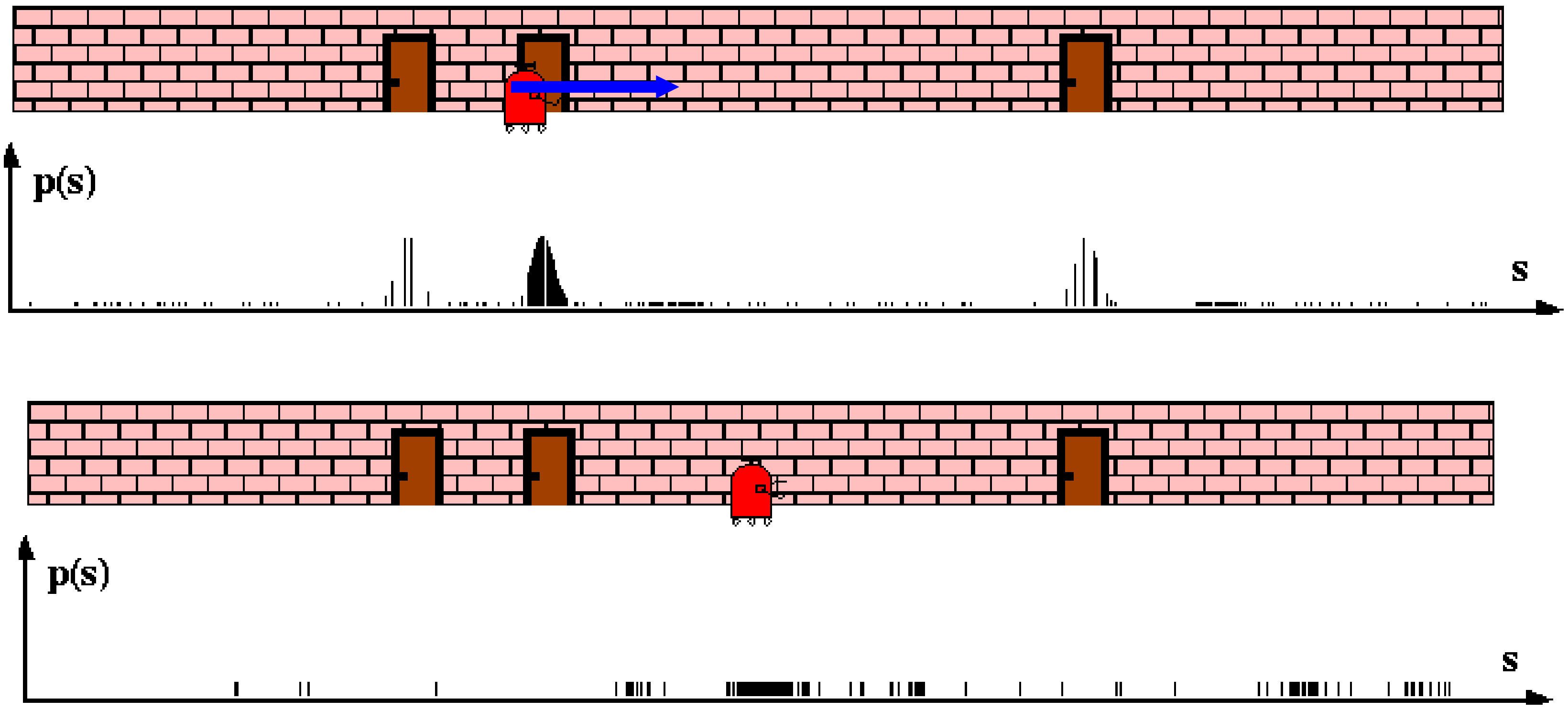
$$Bel(x) \leftarrow \alpha\ p(z \mid x)\ Bel^{-}(x)$$

$$w \leftarrow \frac{\alpha\ p(z \mid x)\ Bel^{-}(x)}{Bel^{-}(x)} = \alpha\ p(z \mid x)$$



Robot Motion

$$Bel^-(x) \leftarrow \int p(x \mid u, x')\, Bel(x')\ \mathrm{d}x'$$



Project Minerva

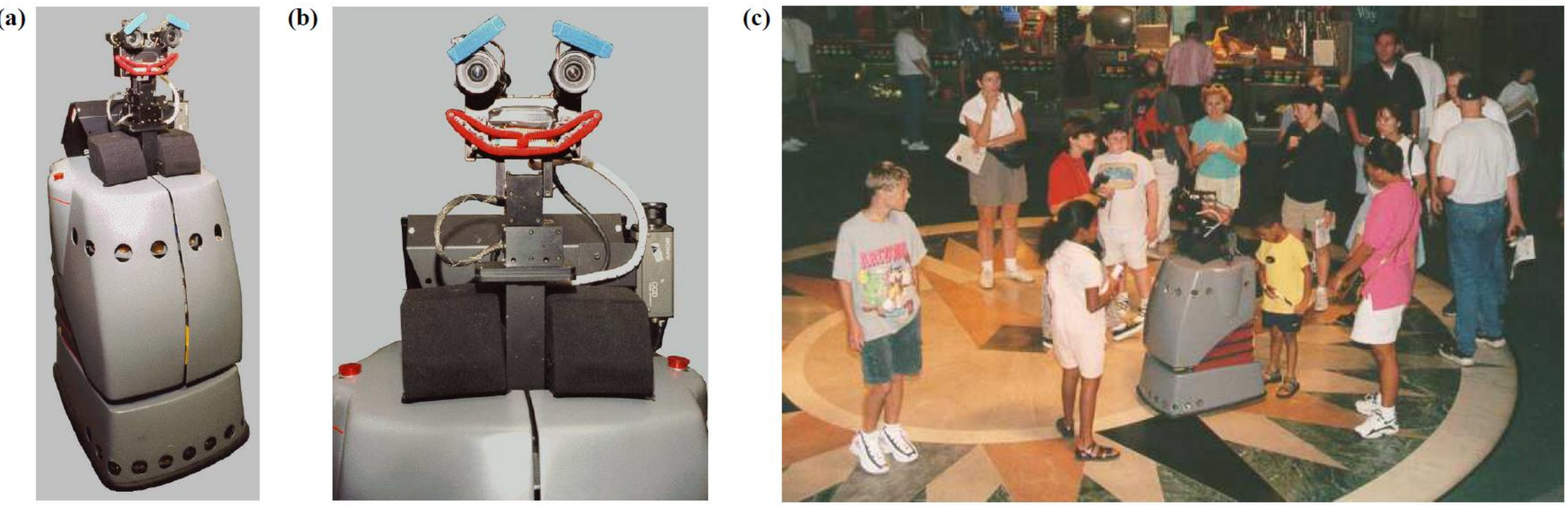
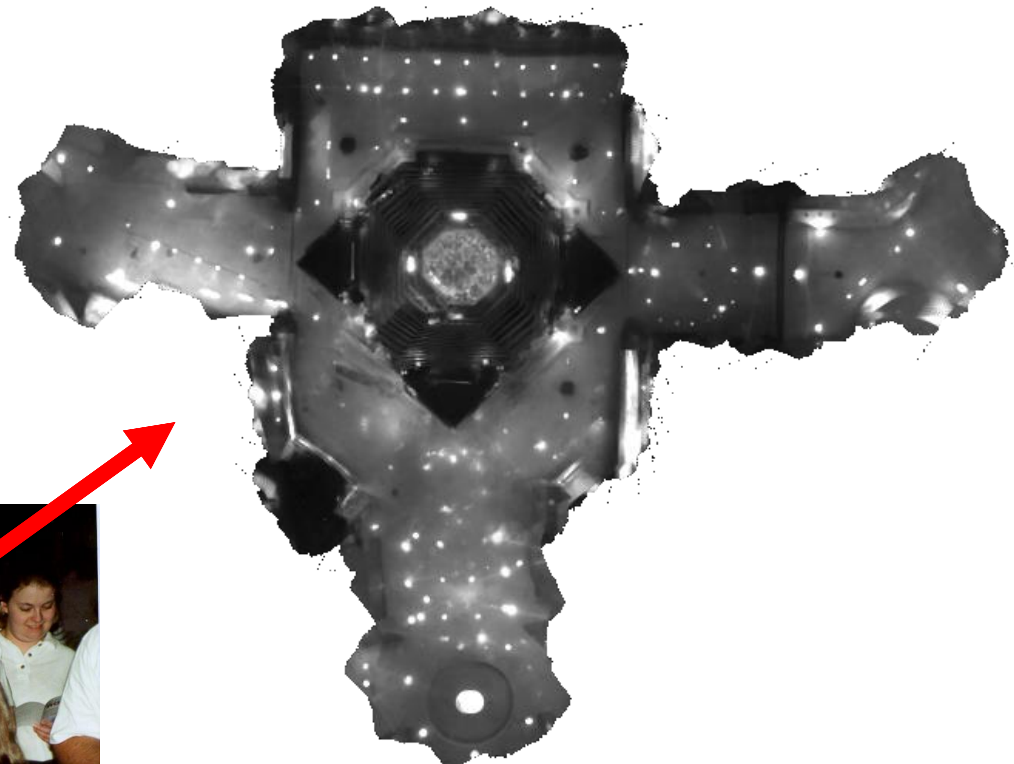
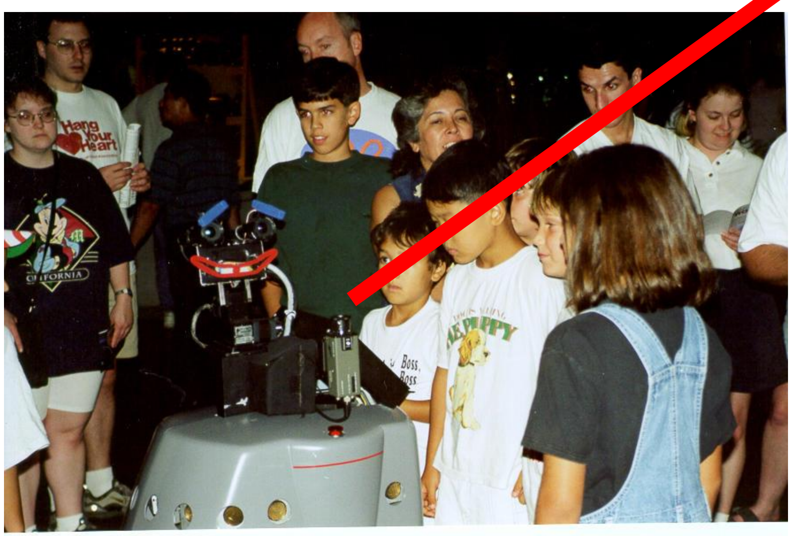


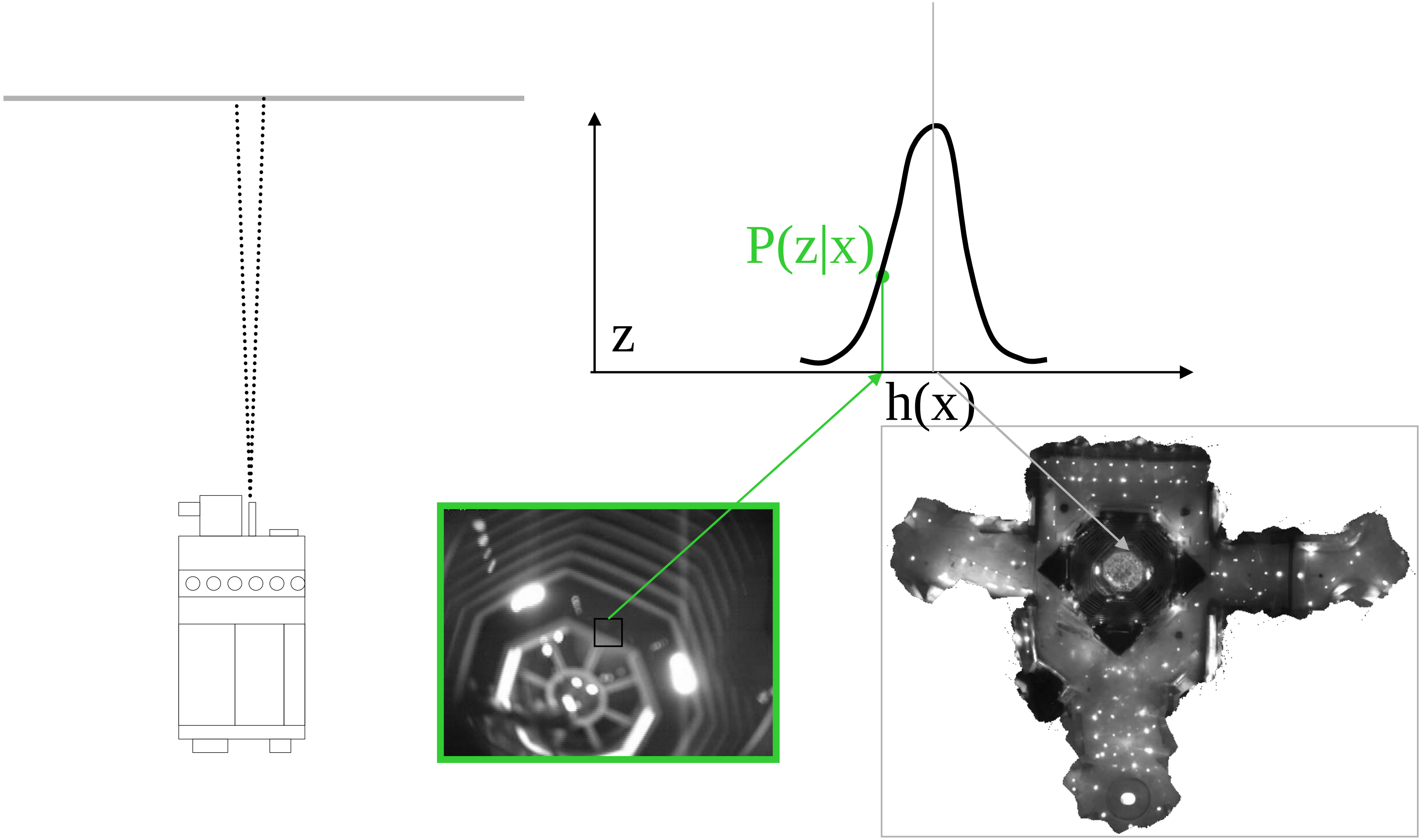
Figure 1: (a) Minerva. (b) Minerva's motorized face. (c) Minerva gives a tour in the Smithsonian's National Museum of American History.

Using Ceiling Maps for Localization



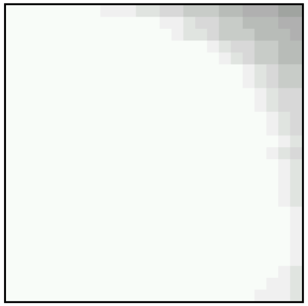


Vision-based Localization

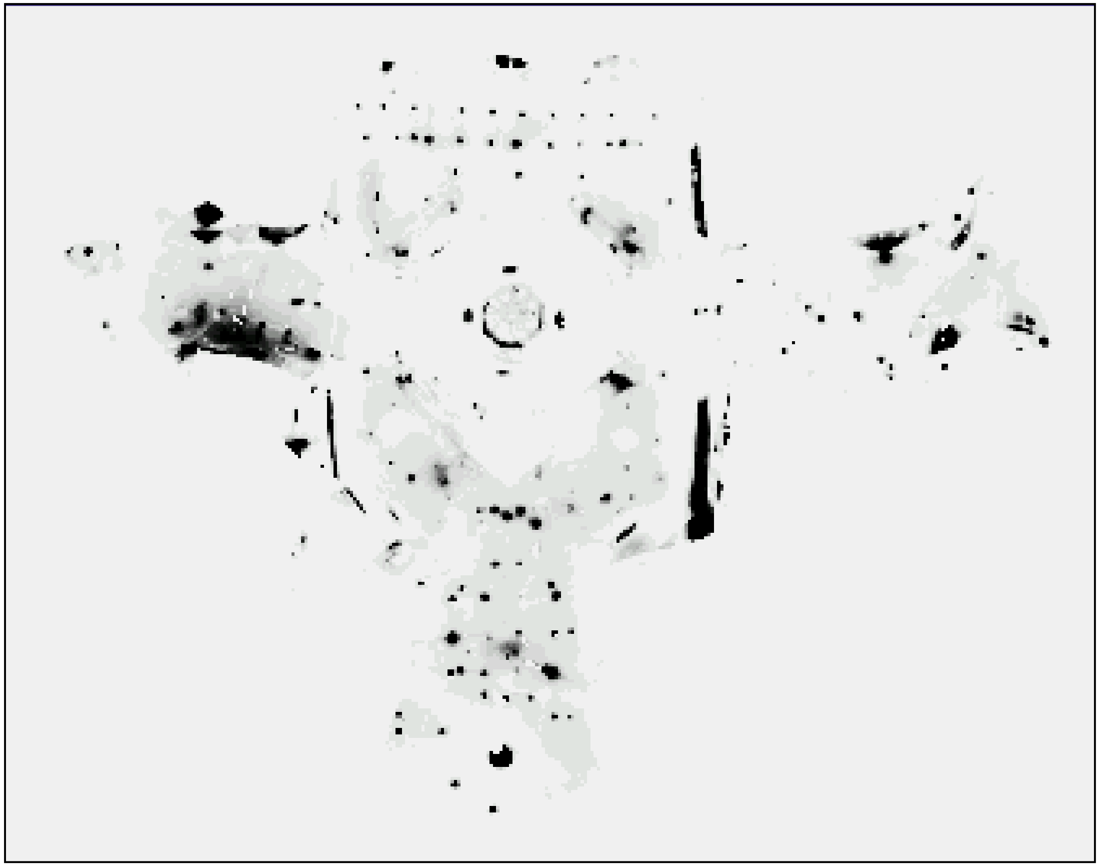


Under a Light

Measurement z:

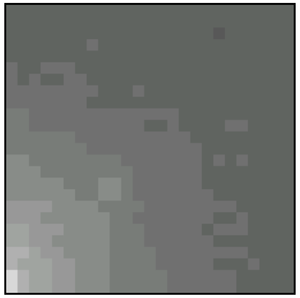


***P(z|x)*:**

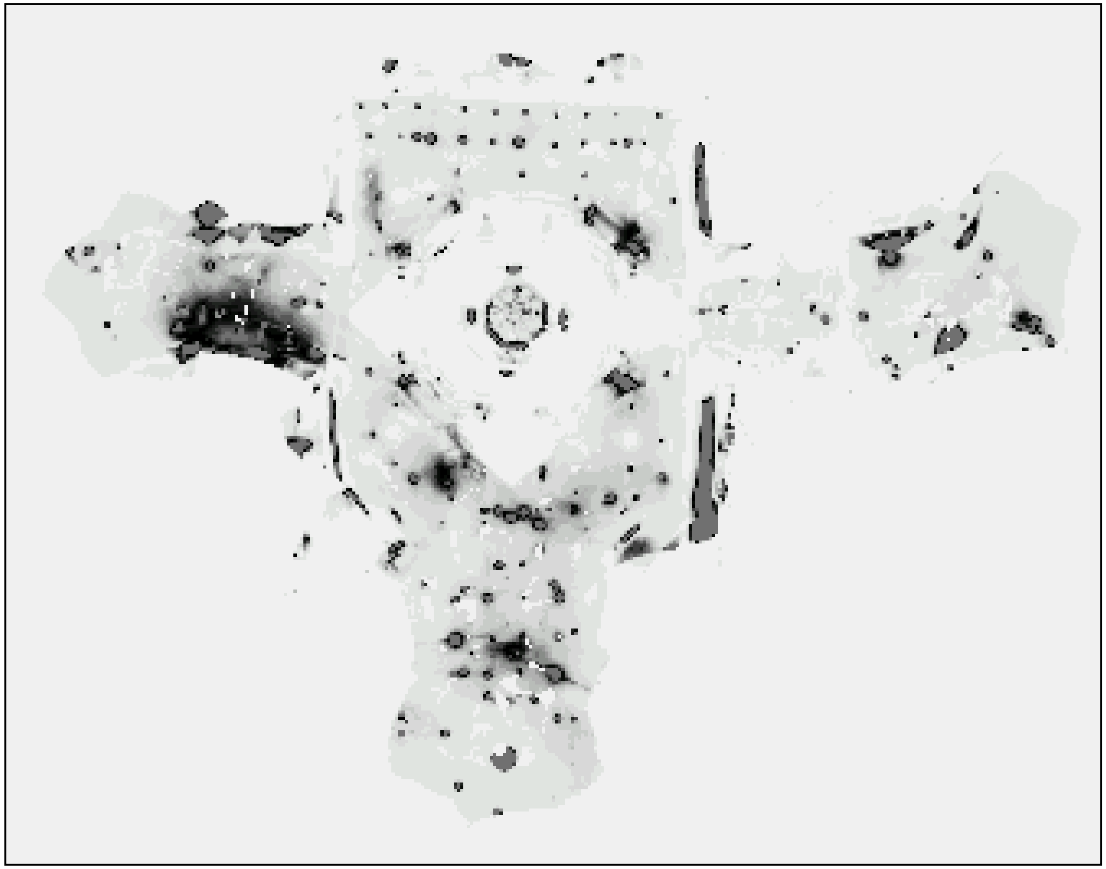


Next to a Light

Measurement z:



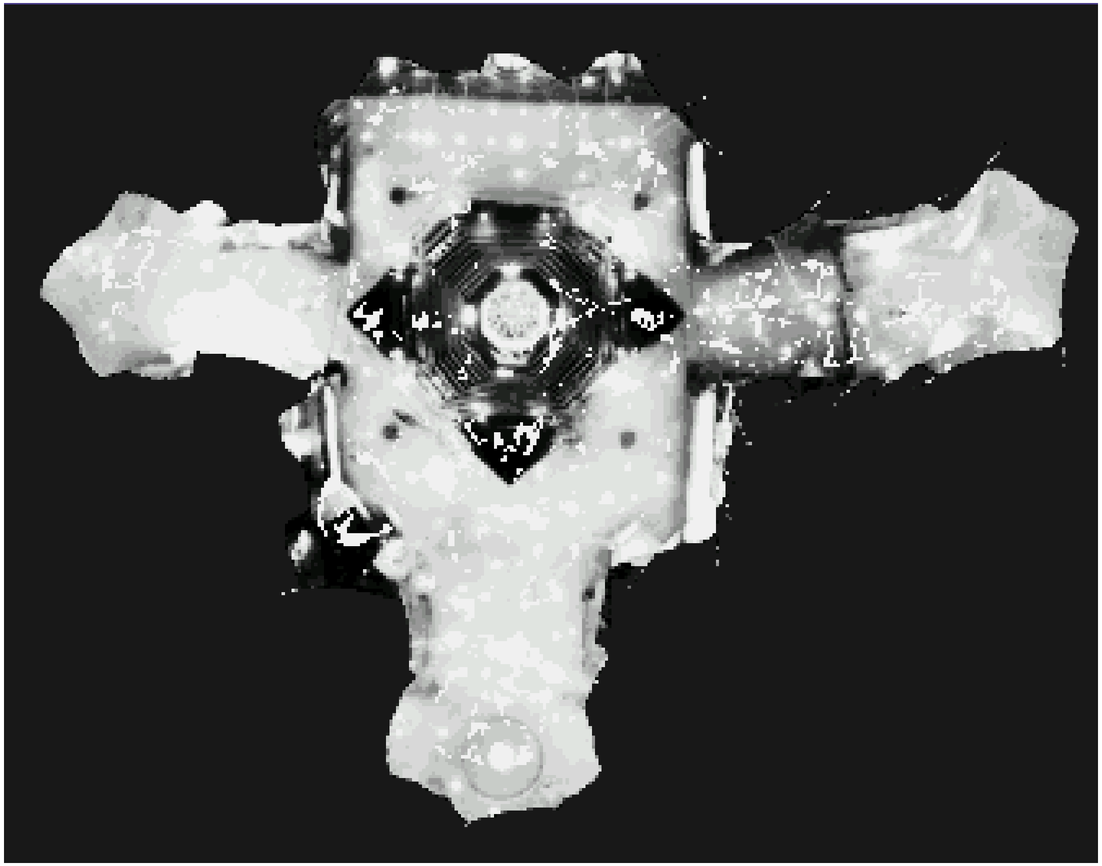
***P(z|x)*:**



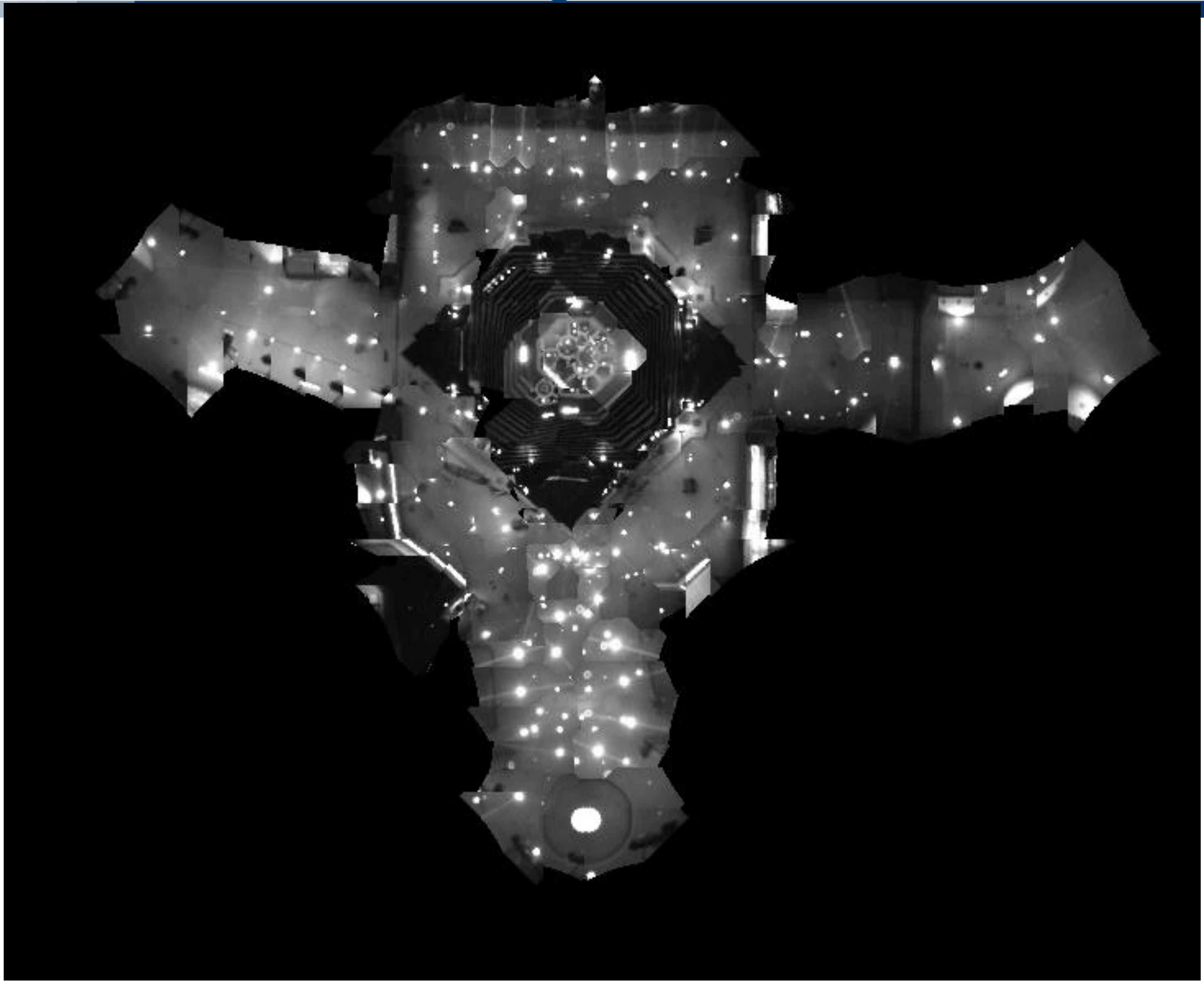
Elsewhere

Measurement z:

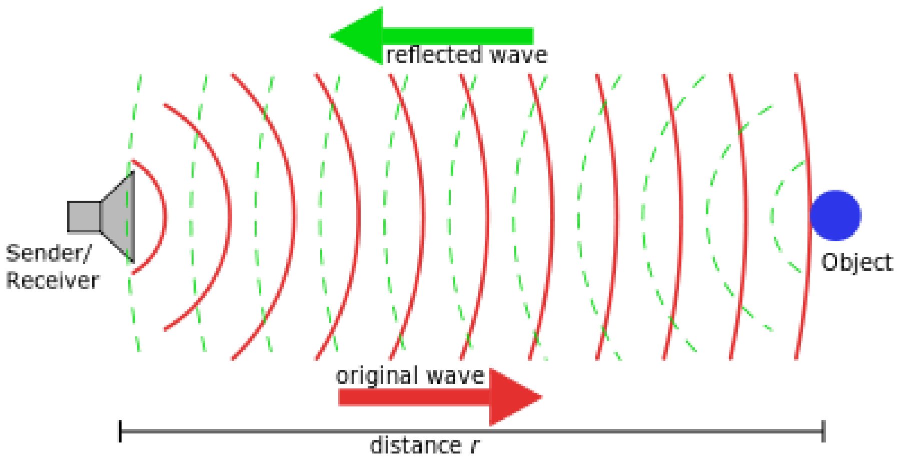
***P(z|x)*:**

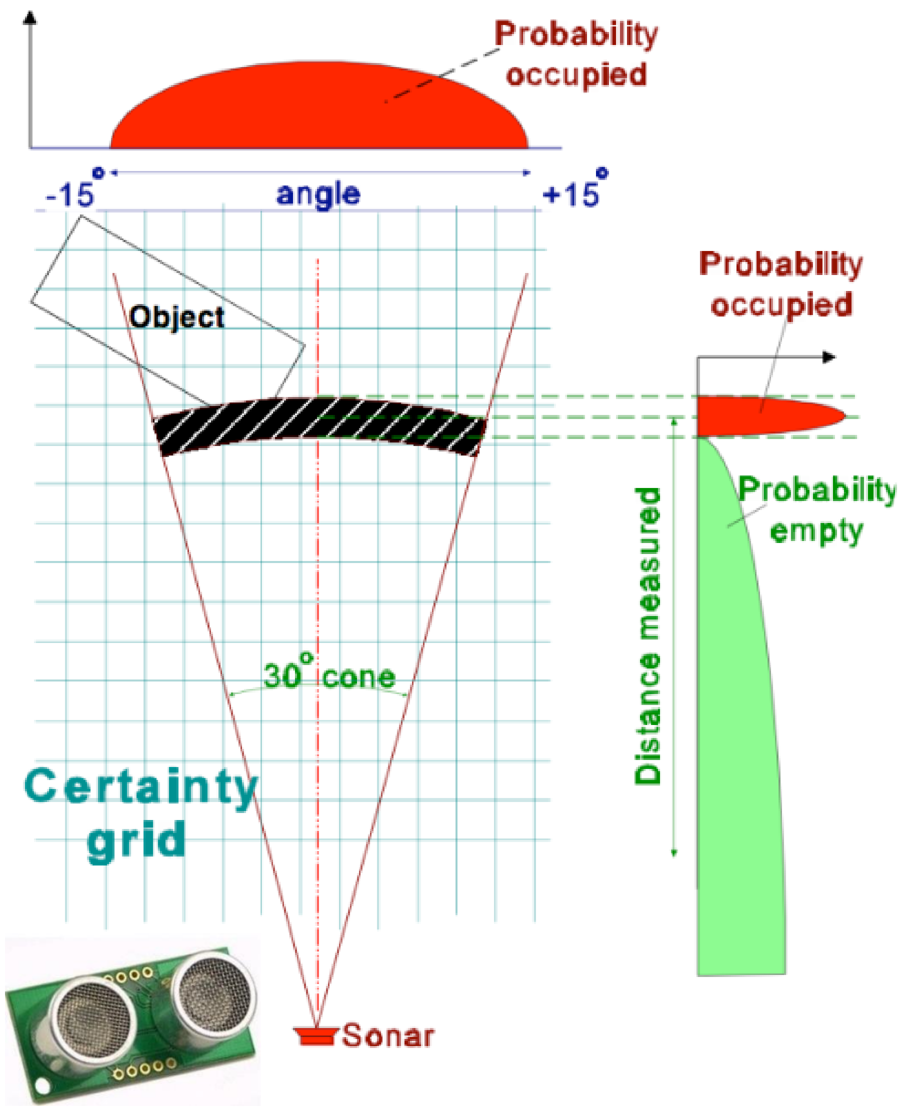


Global Localization Using Vision



Sonar Sensor Model (Ultrasound Wave)





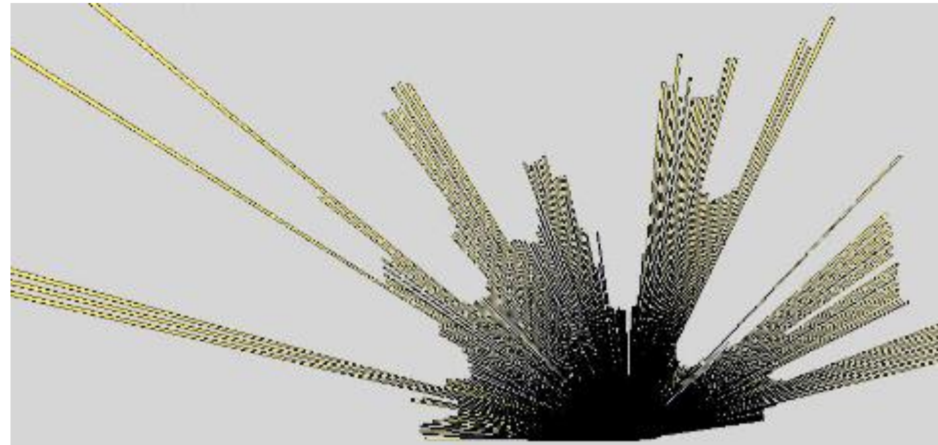
An US wave is sent by a transducer

- Time of flight is measured
- Distance is computed from it
- Obstacle could be anywhere on the arc at distance D
- The space closer than D is likely to be free.

Laser Range Finder Sensor Model

Lasers are definitely more accurate sensors

- 180 ranges over 180°
  (up to 360 in some models)
- 1 to 64 planes scanned
- 10-75 scans/second
- <1cm range resolution
- Max range up to 50-80 m
- Problems only with mirrors, glass, and matte bla



< 1000 €

~ 6000 €

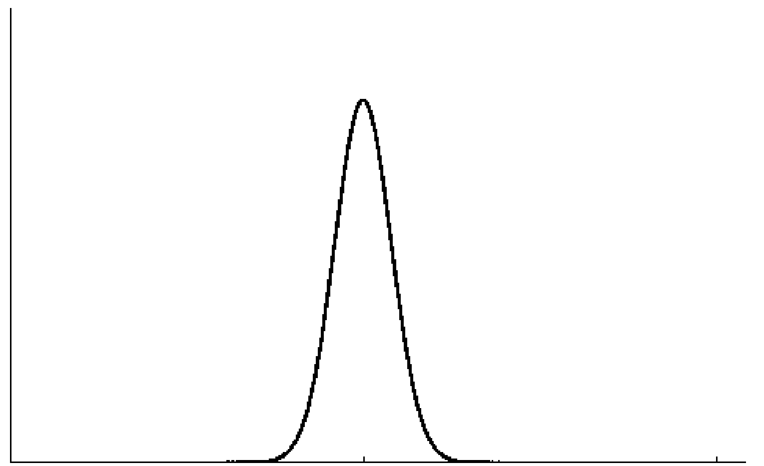
~ 40.000 €

> 80.000 €

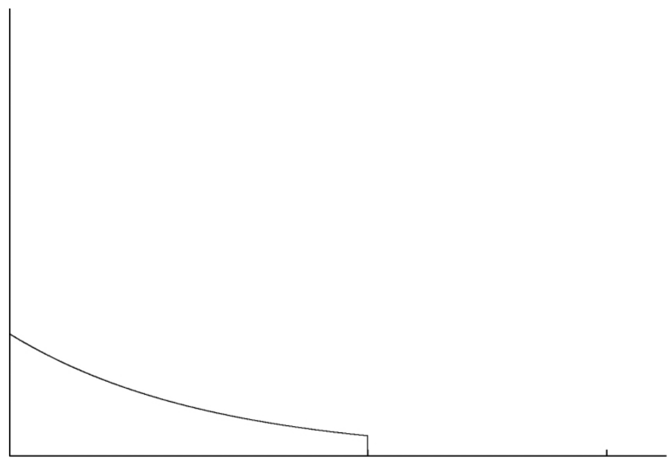
Beam Based Sensor Model (I)

The laser range finder model describe each single measurement using

Measurement noise



Unexpected obstacles

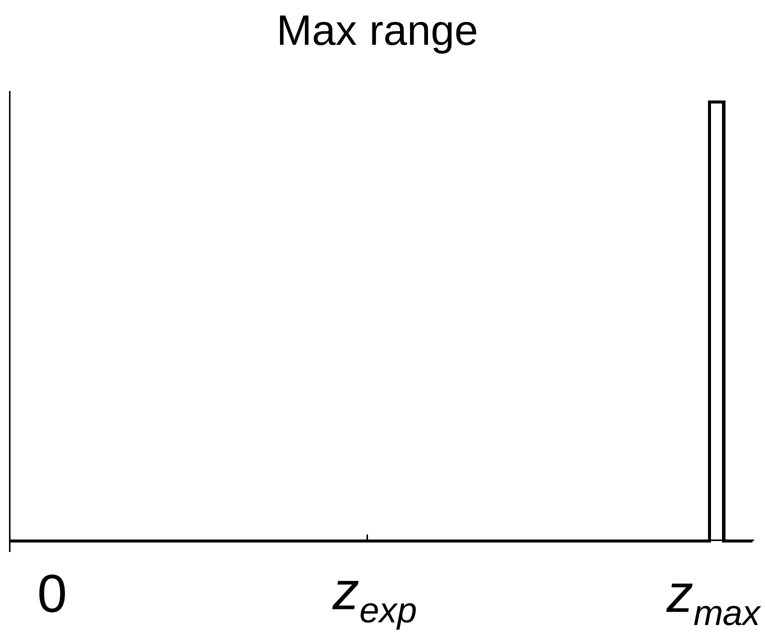


$$P_{hit}(z \mid x, m) = \eta \frac{1}{\sqrt{2\pi b}} e^{-\frac{1}{2}\frac{(z-z_{exp})^2}{b}}$$

$$P_{unexp}(z \mid x, m) = \begin{cases} \eta \, \lambda \, e^{-\lambda z} & z < z_{exp} \\ 0 & otherwise \end{cases}$$

Beam Based Sensor Model (II)

The laser range finder model describe each single measurement using

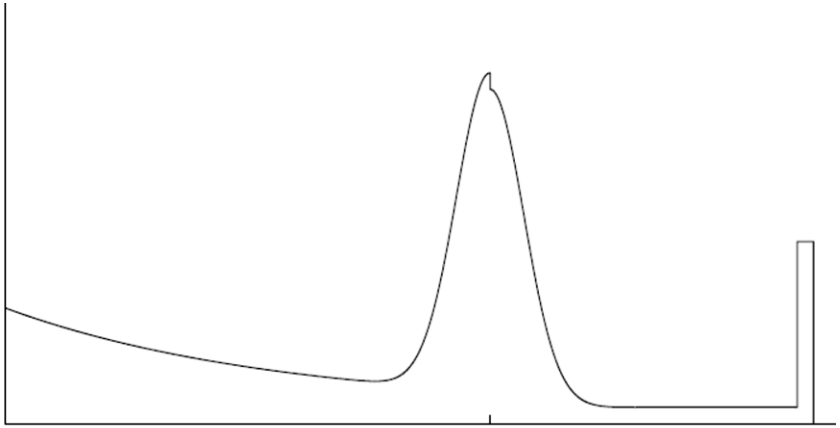


$$P_{rand}(z \mid x, m) = \eta \frac{1}{z_{\max}}$$

$$P_{\max}(z \mid x, m) = \eta \frac{1}{z_{small}}$$

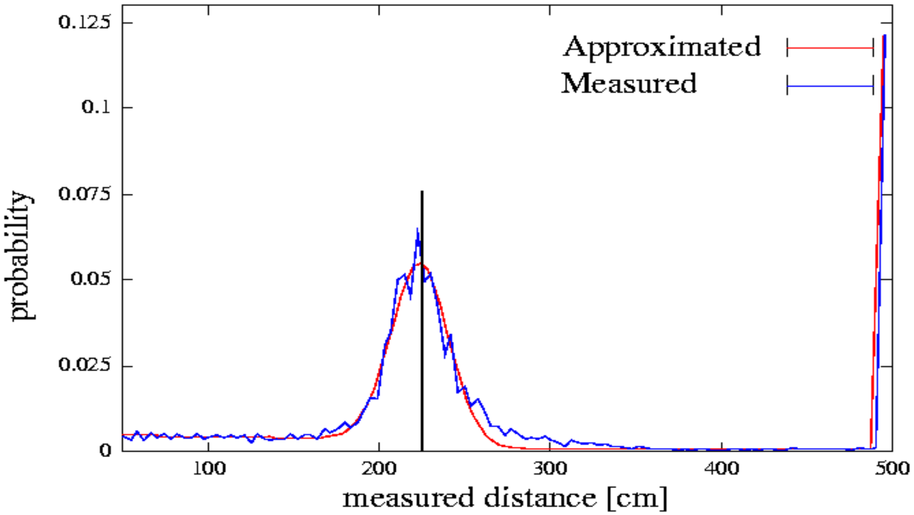
Beam Based Sensor Model (III)

The laser range finder model describe each single measurement using



$$P(z \mid x, m) = \begin{pmatrix} \alpha_{\text{hit}} \\ \alpha_{\text{unexp}} \\ \alpha_{\text{max}} \\ \alpha_{\text{rand}} \end{pmatrix}^T \cdot \begin{pmatrix} P_{\text{hit}}(z \mid x, m) \\ P_{\text{unexp}}(z \mid x, m) \\ P_{\text{max}}(z \mid x, m) \\ P_{\text{rand}}(z \mid x, m) \end{pmatrix}$$

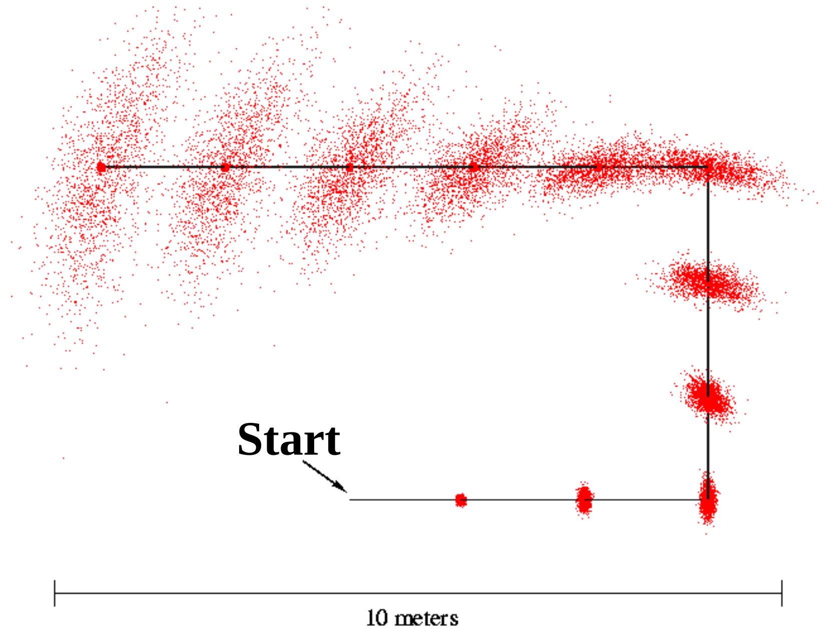
Which, in practice, turns out to be quite accurate!



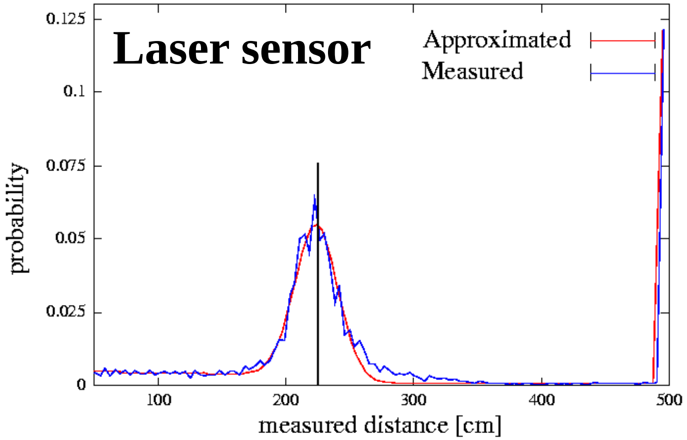
Matteo Matteucci – matteo.matteucci@polimi.it

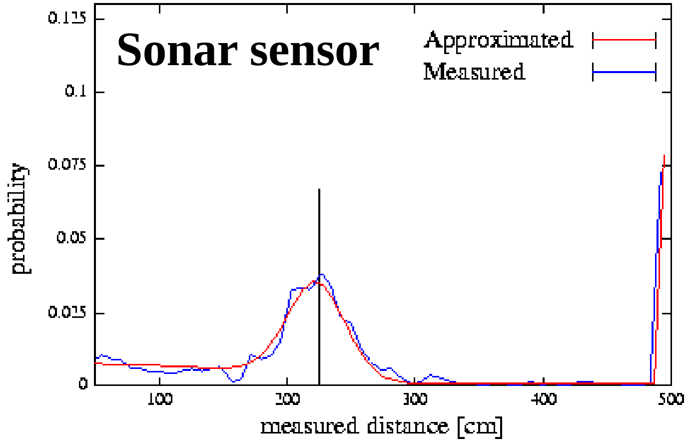
Monte Carlo Localization with Laser

Stochastic motion model

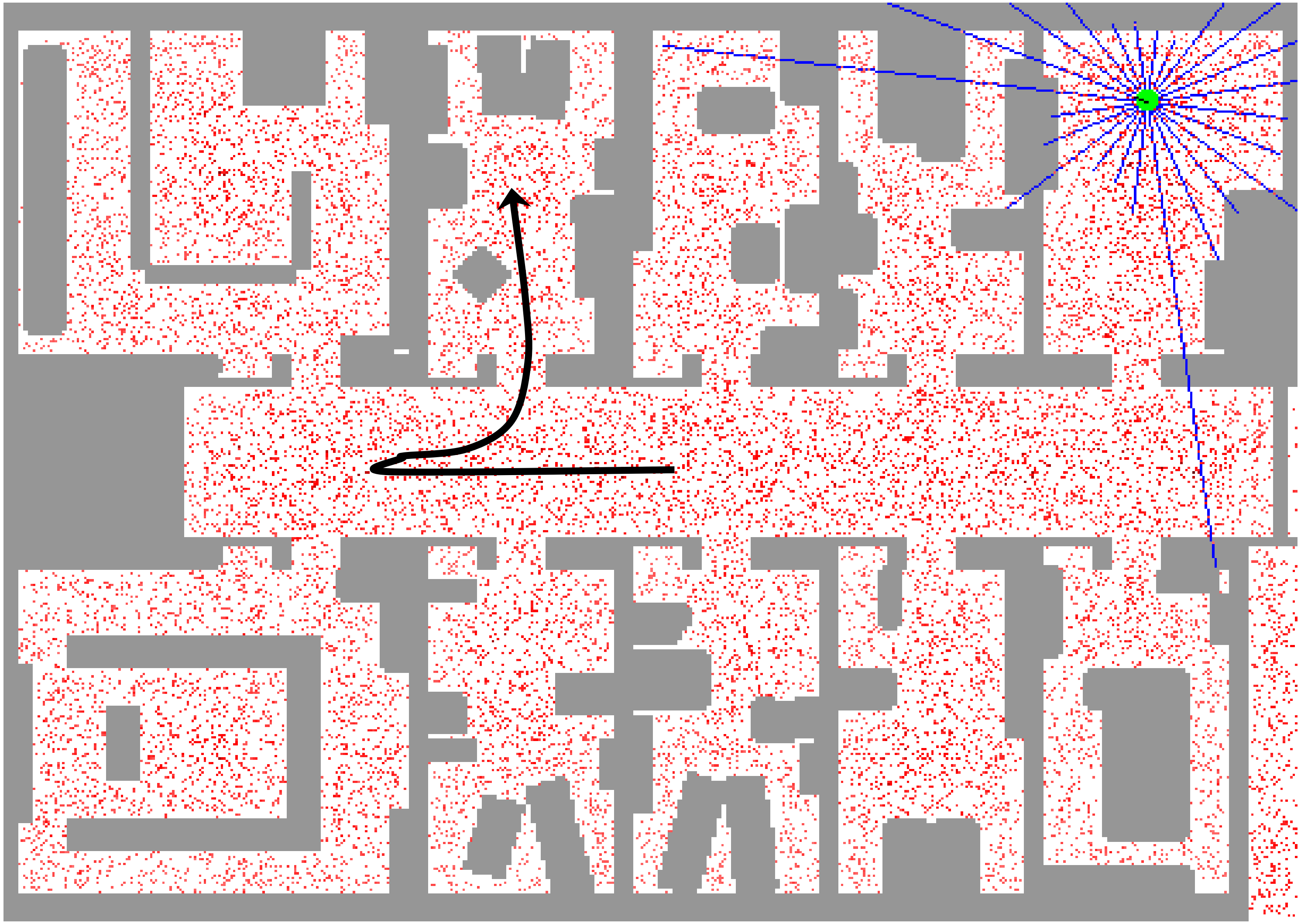


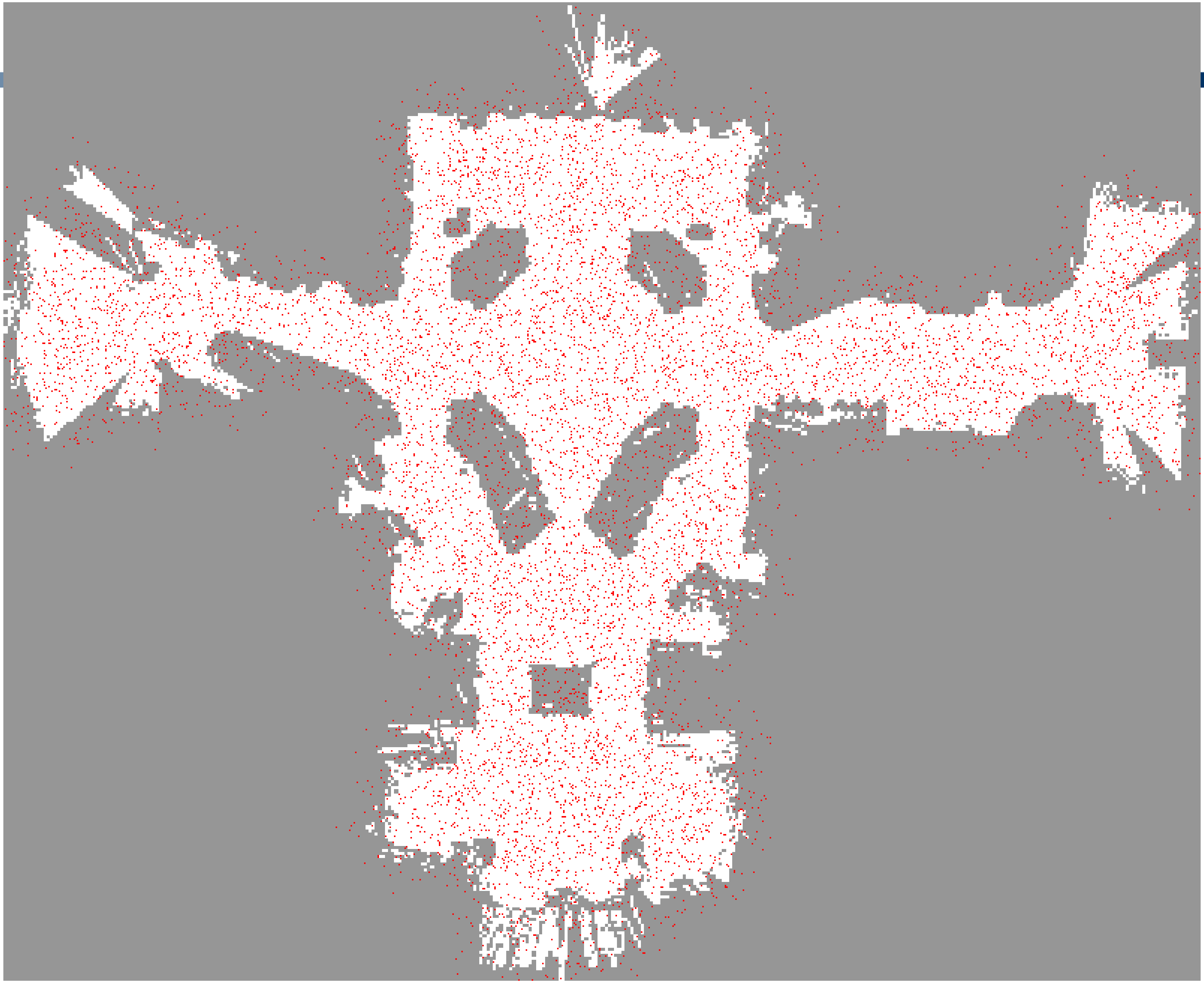
Proximity sensor model

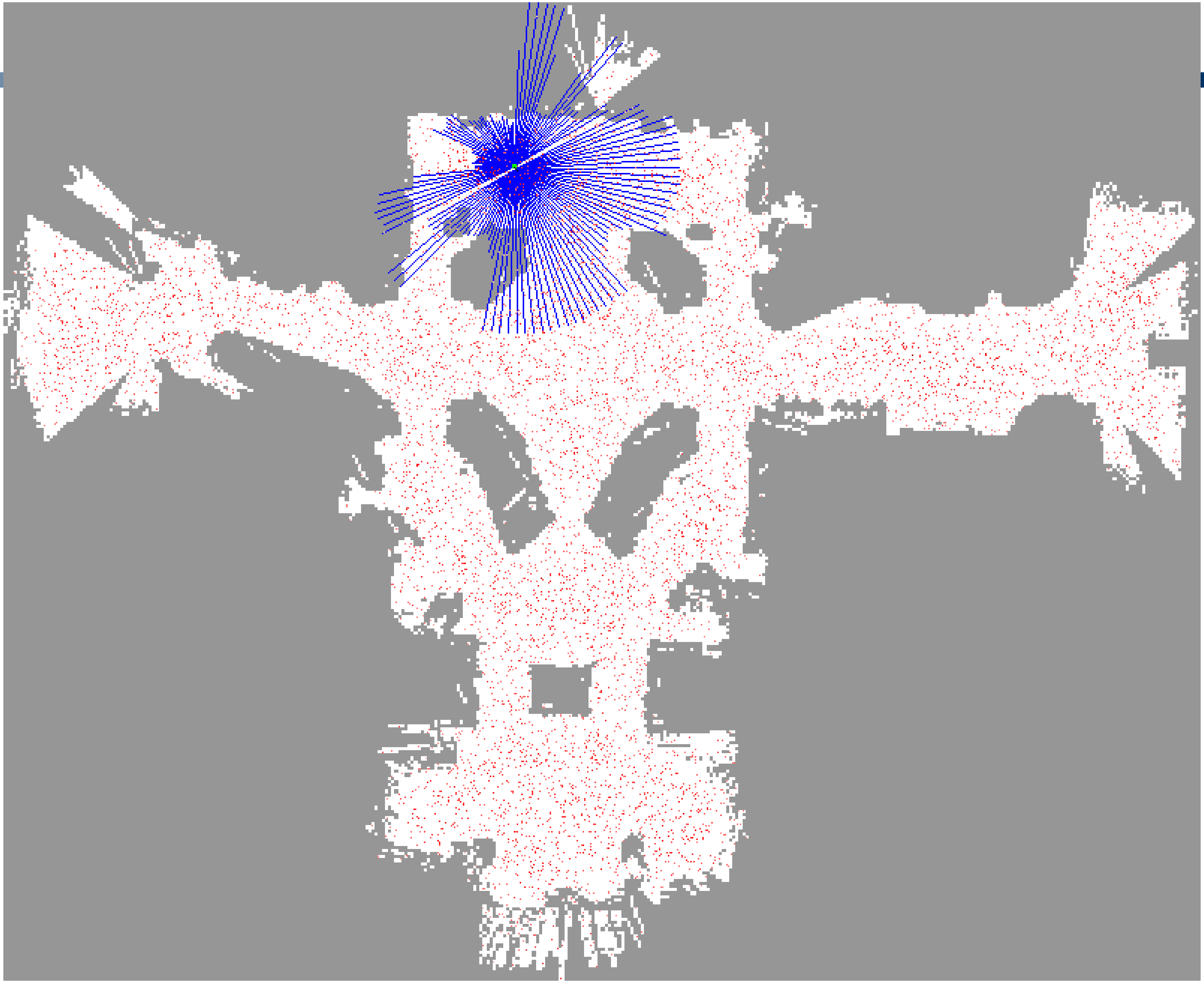


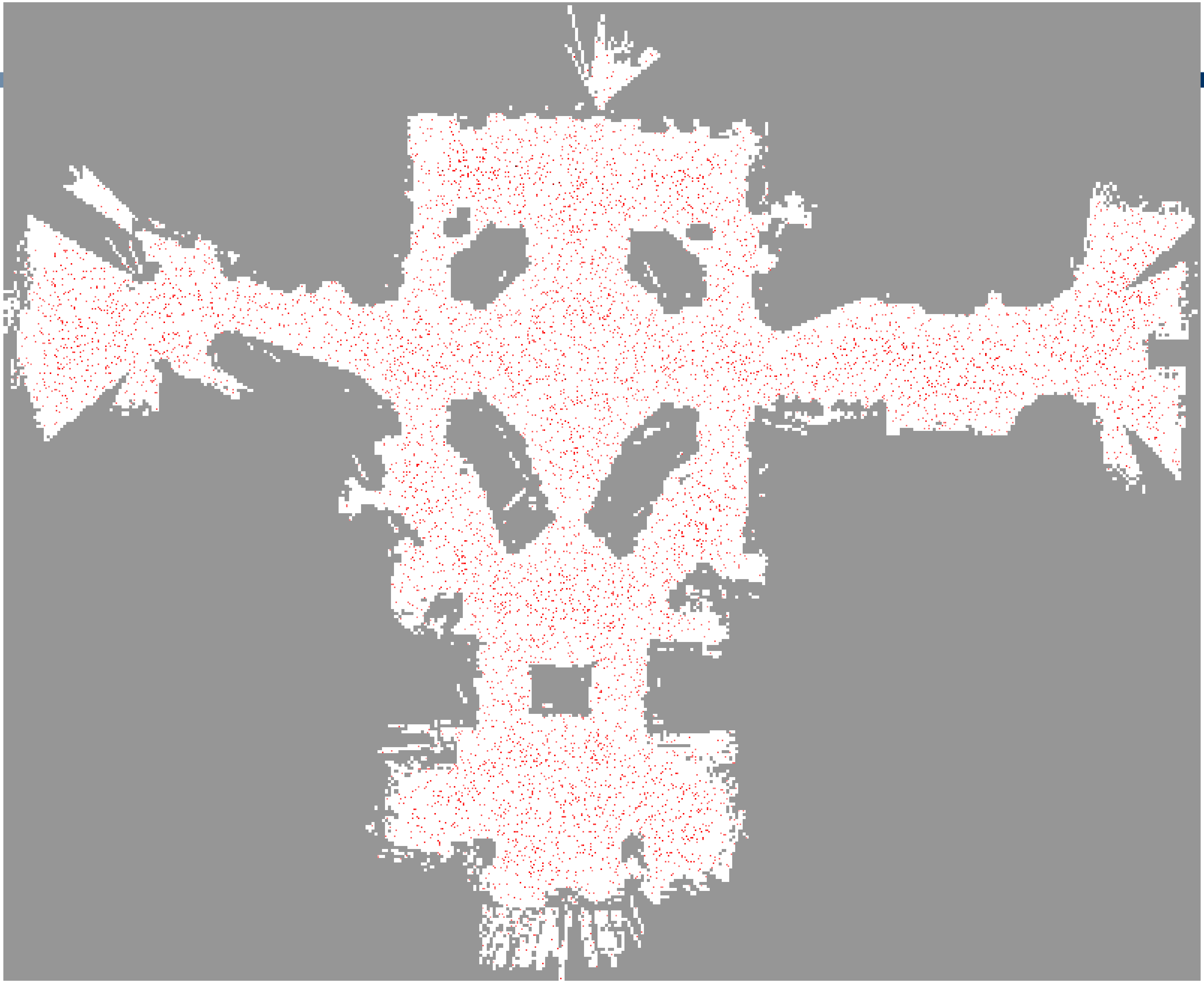


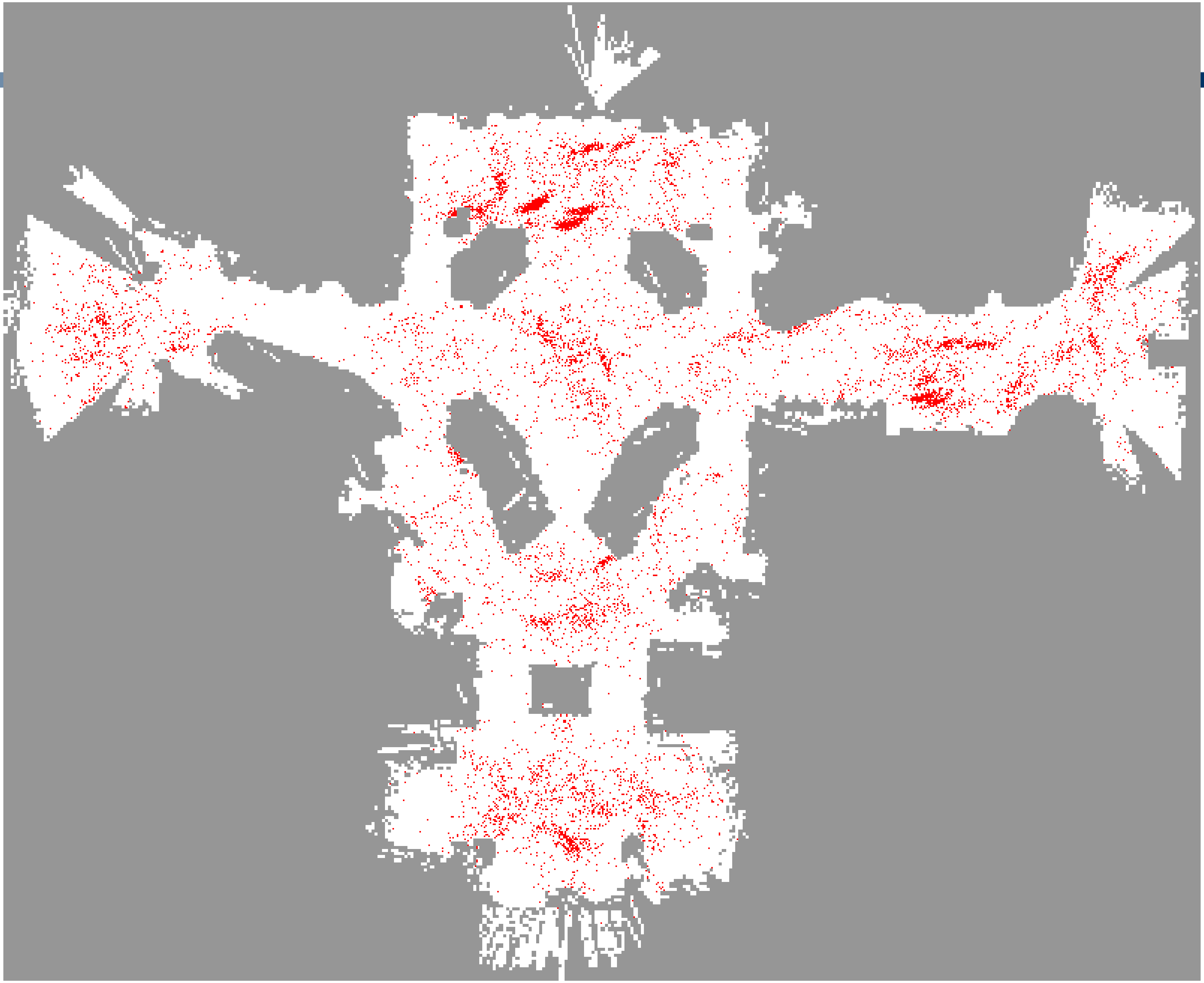
Sample-based Localization (sonar)

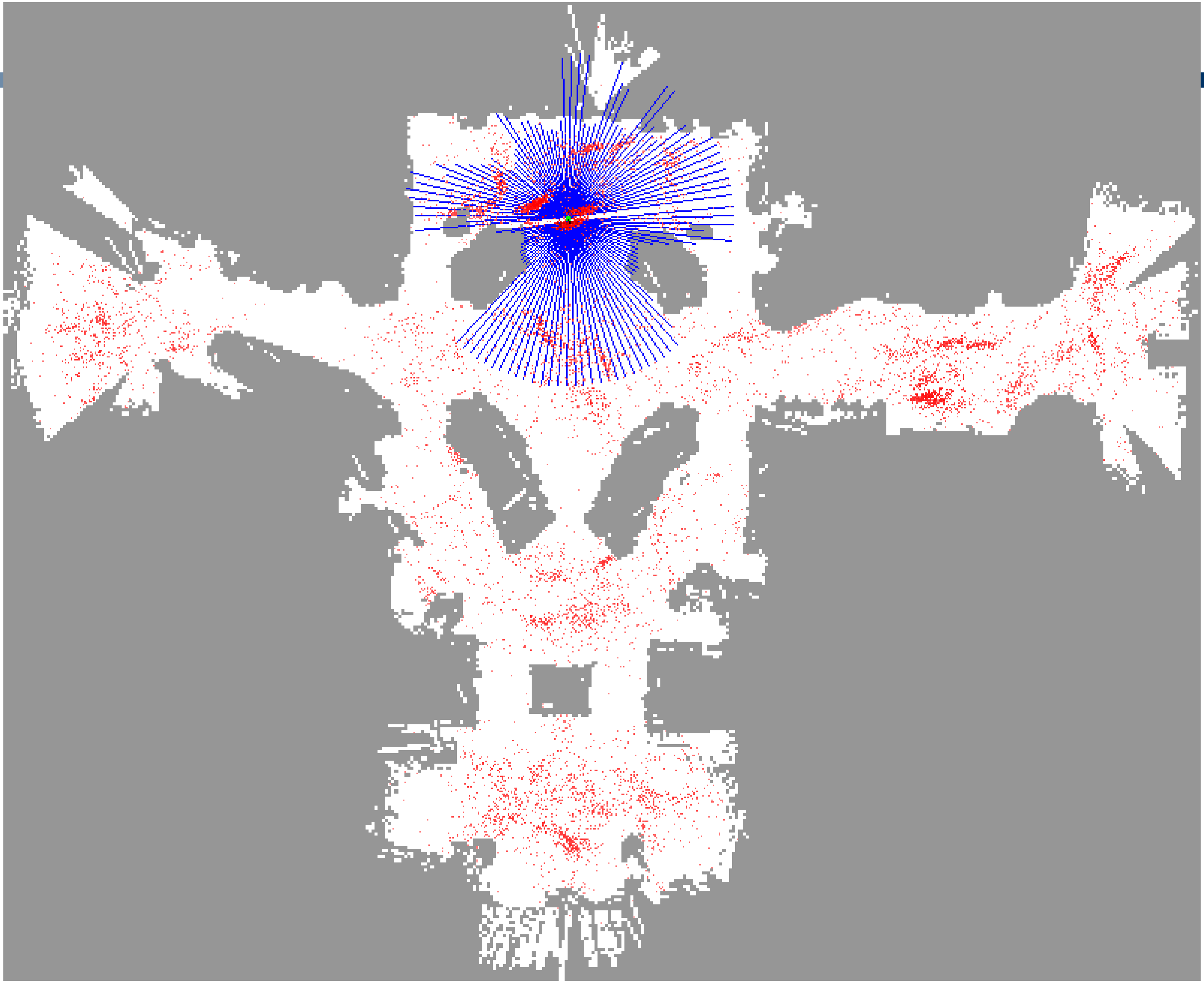


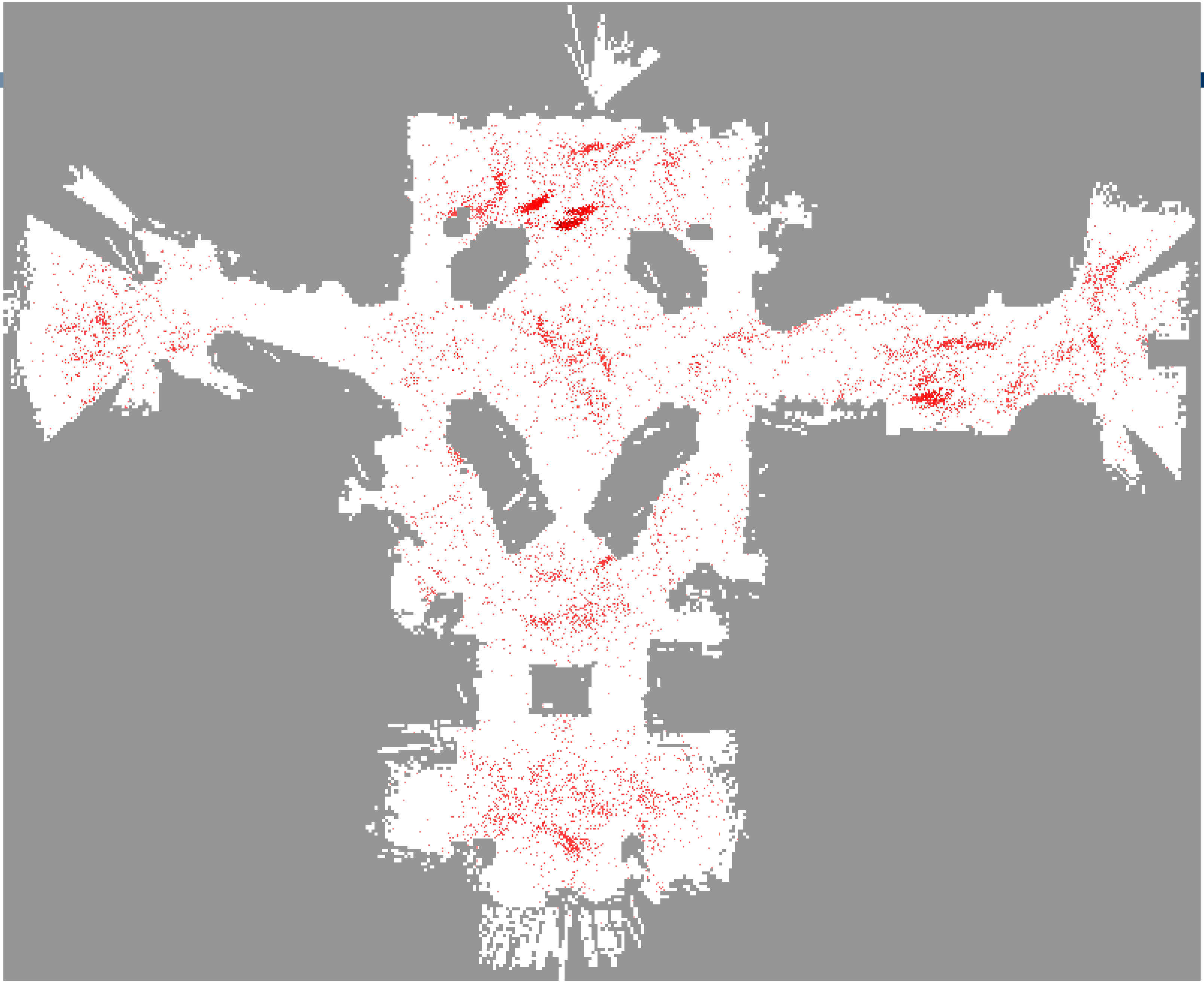


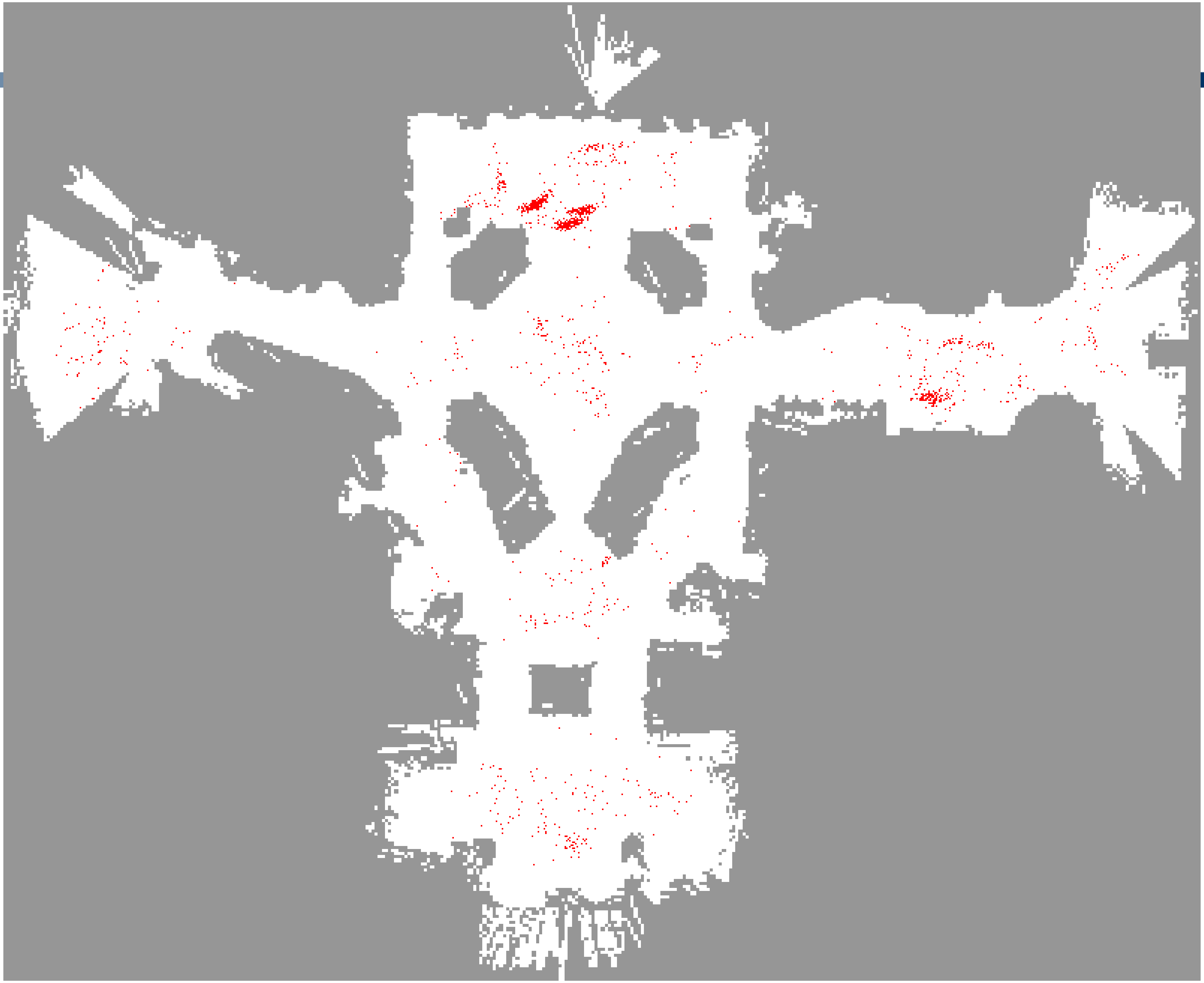


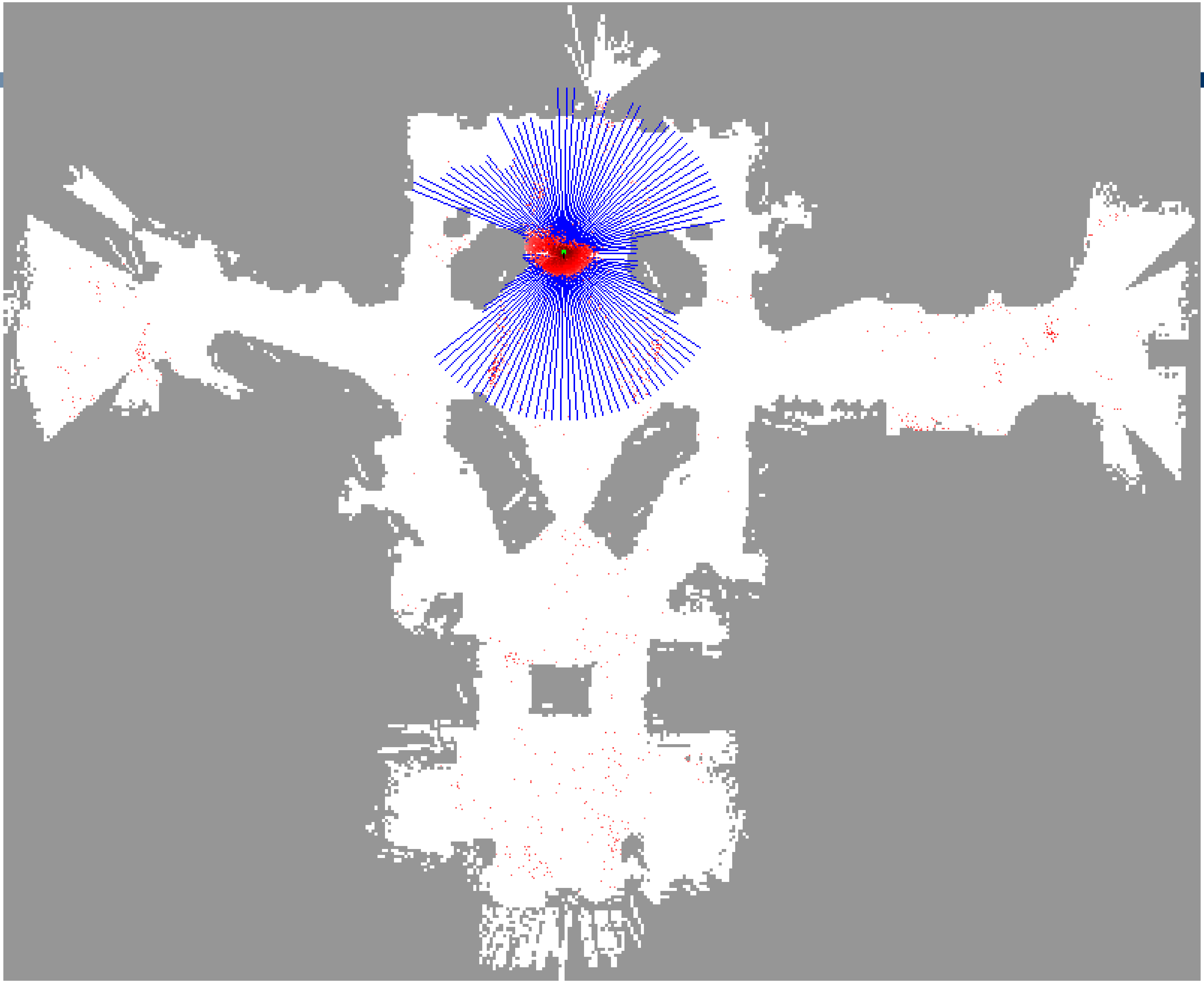


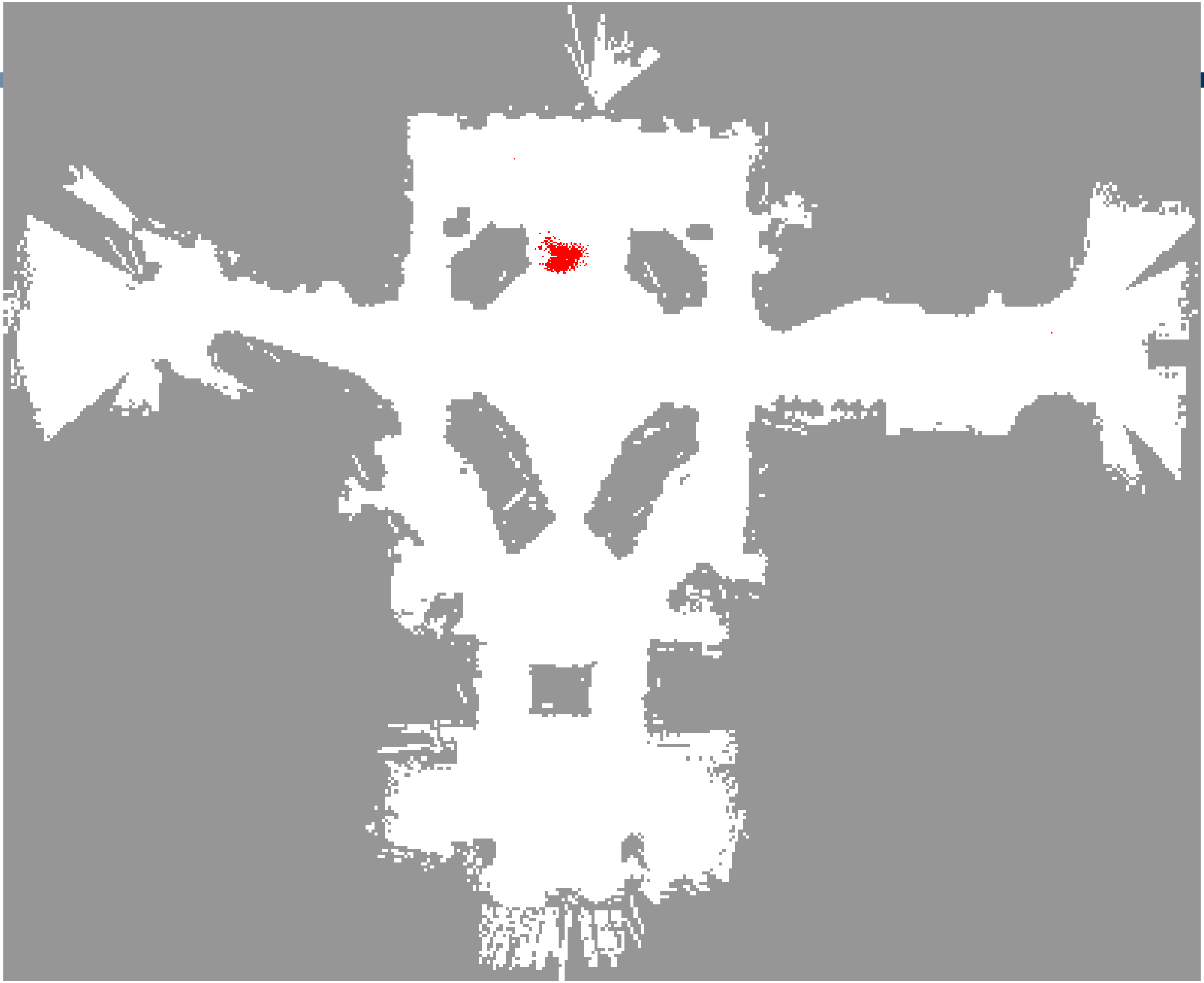


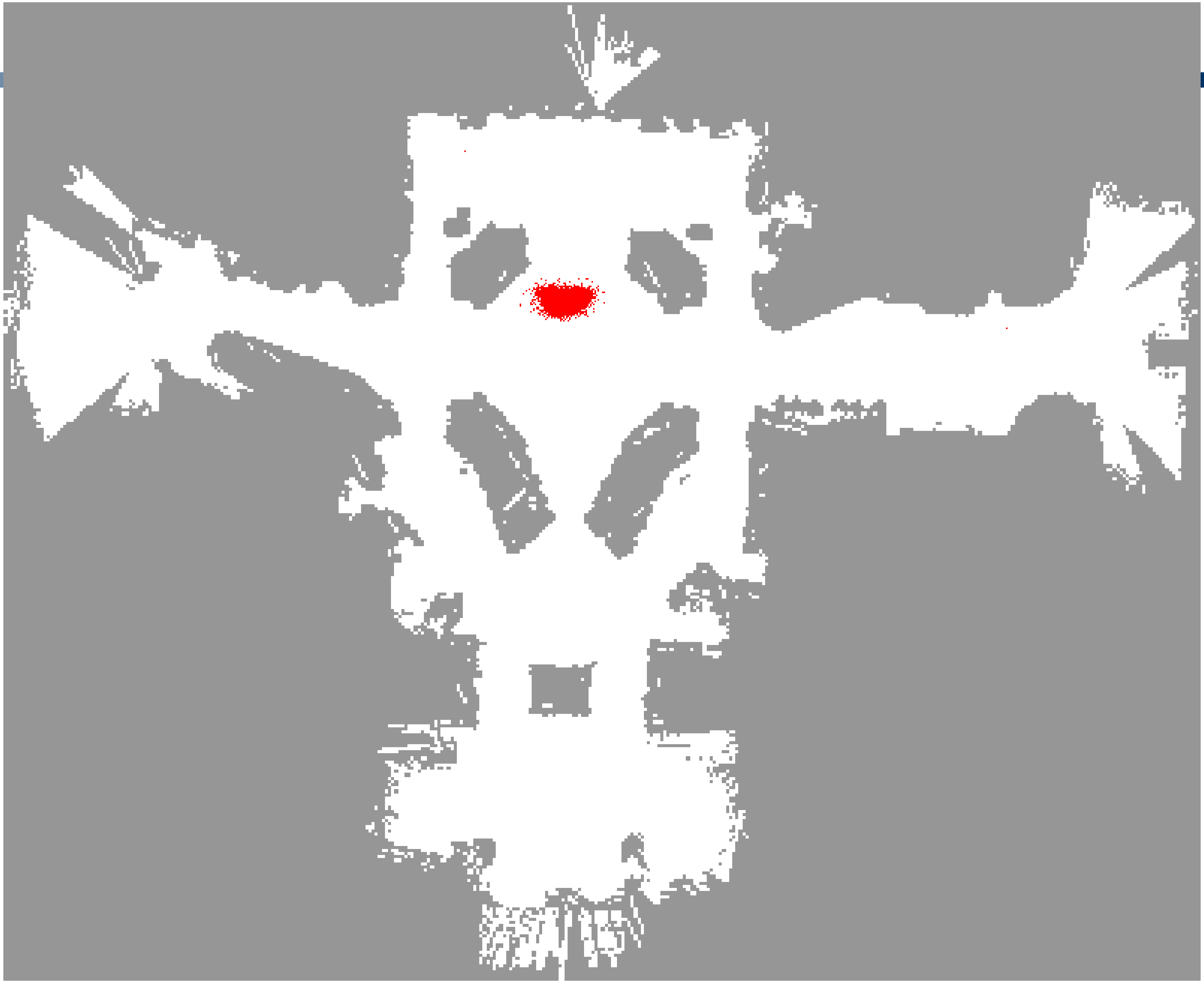


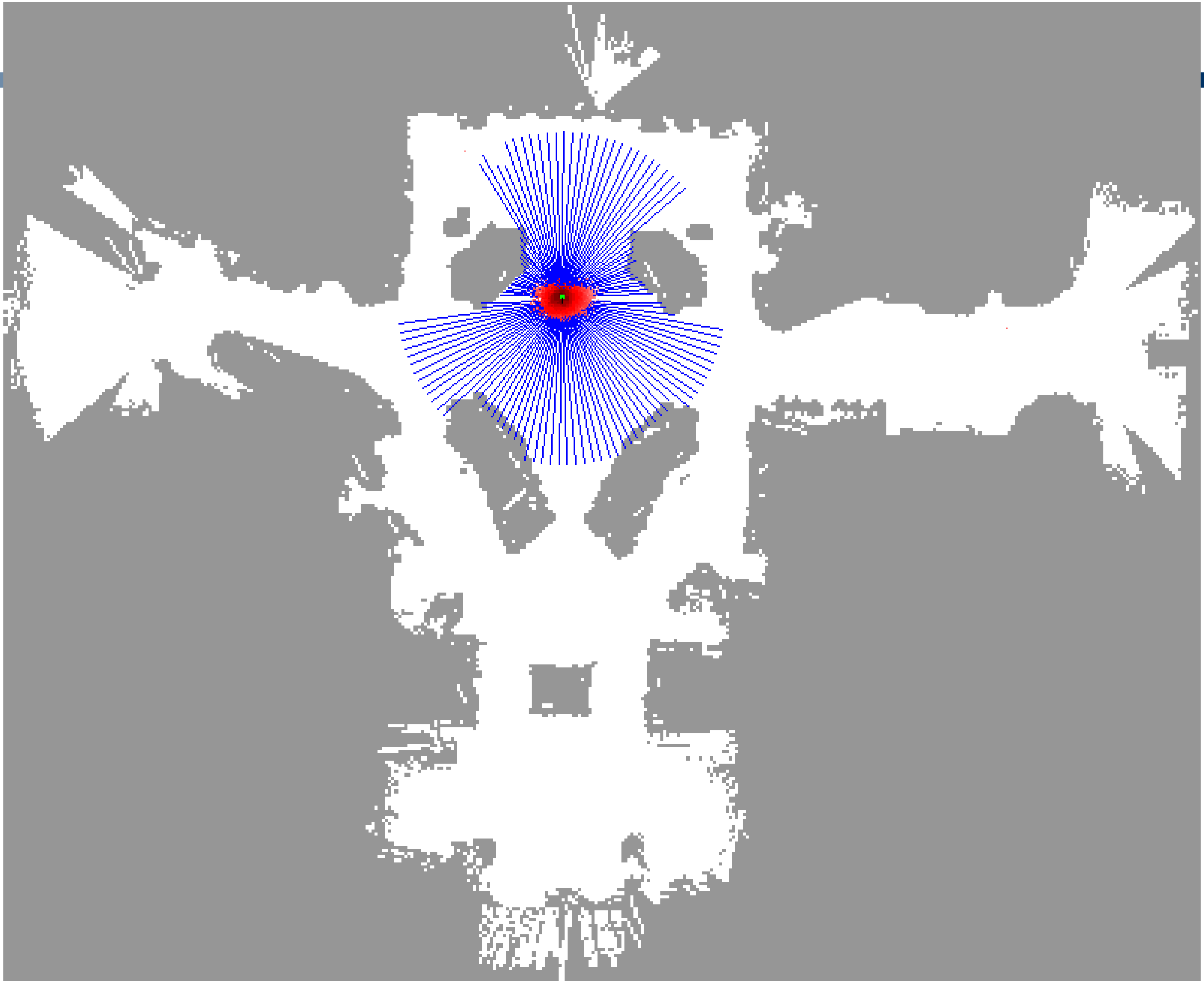


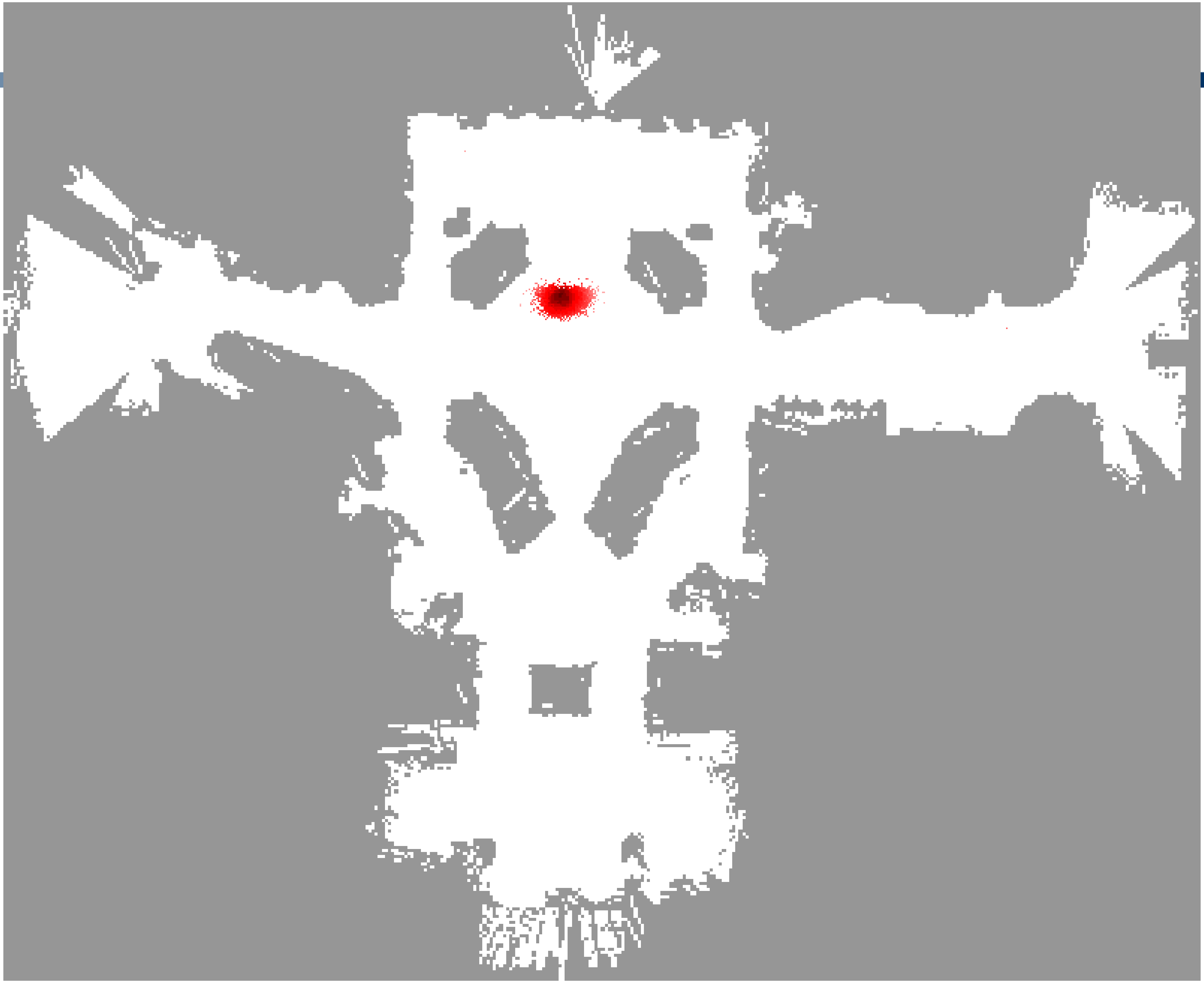


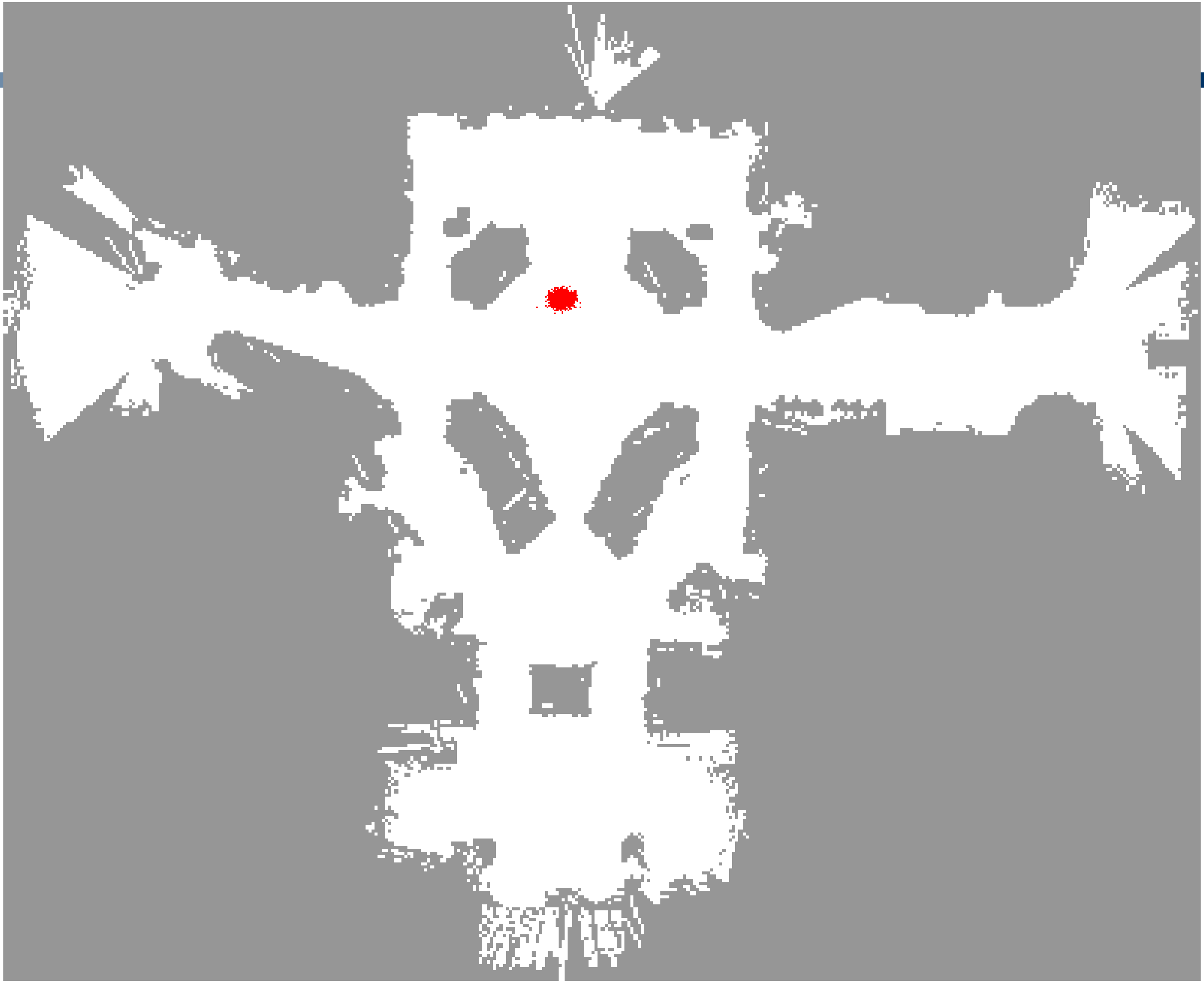


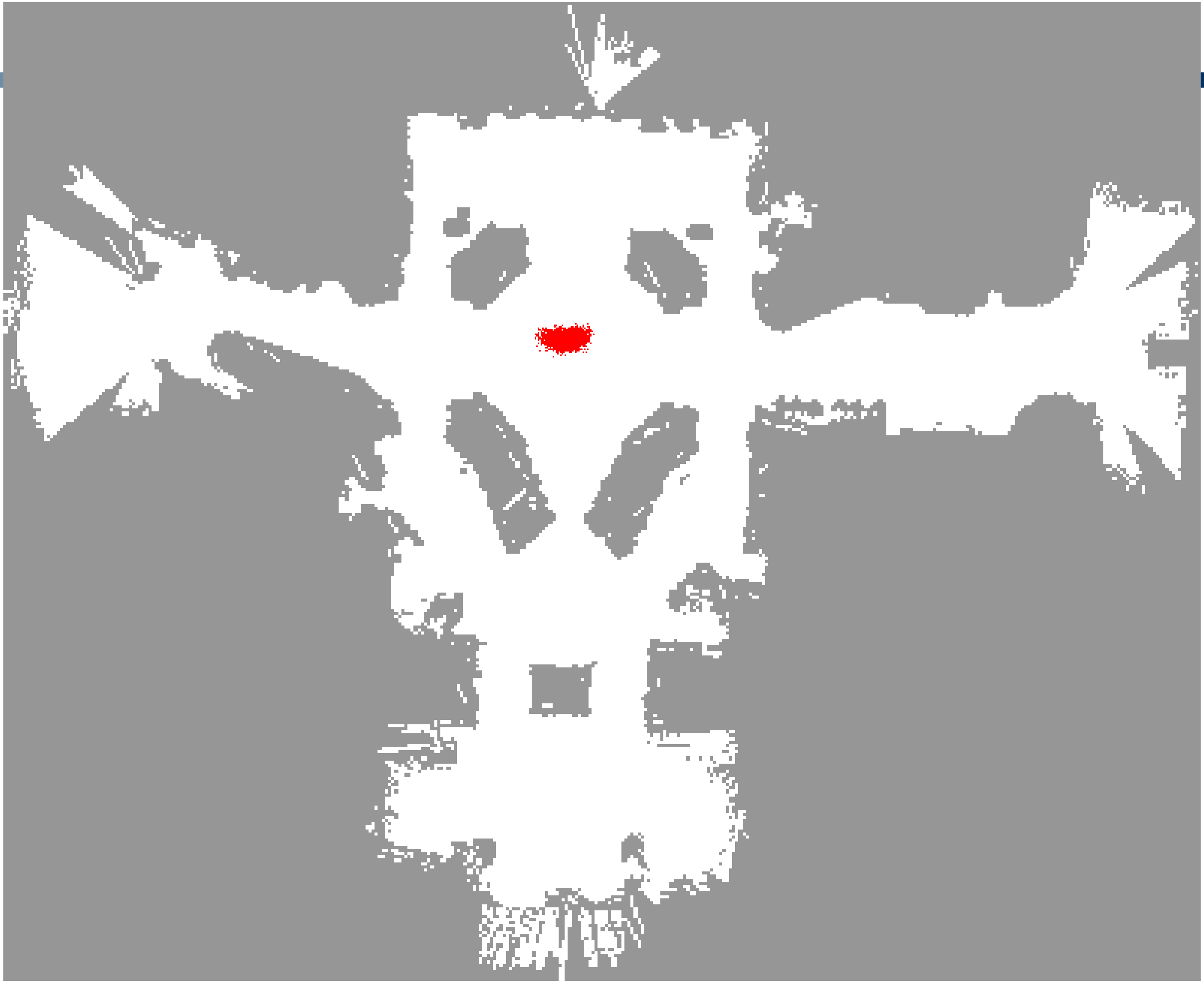


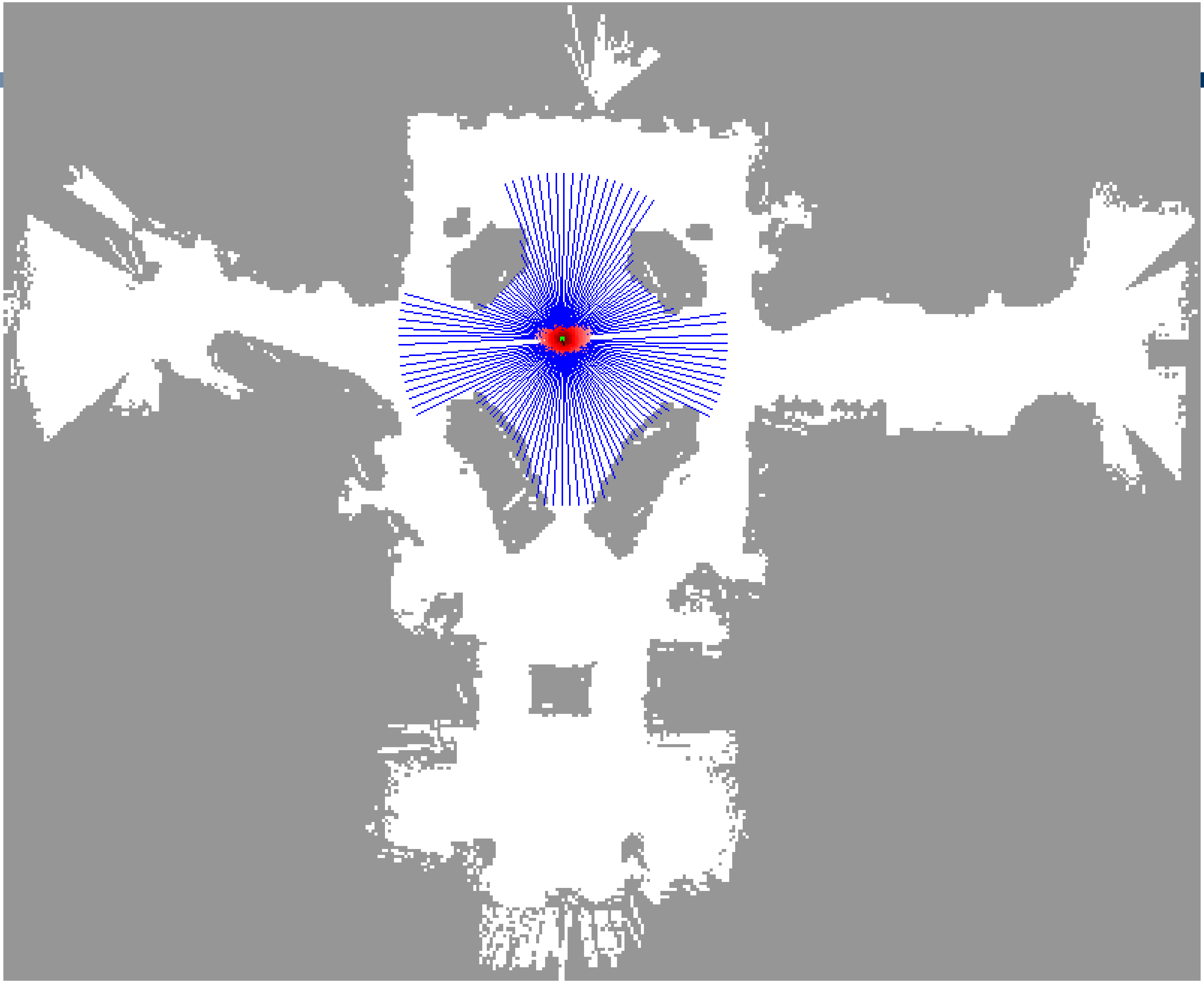


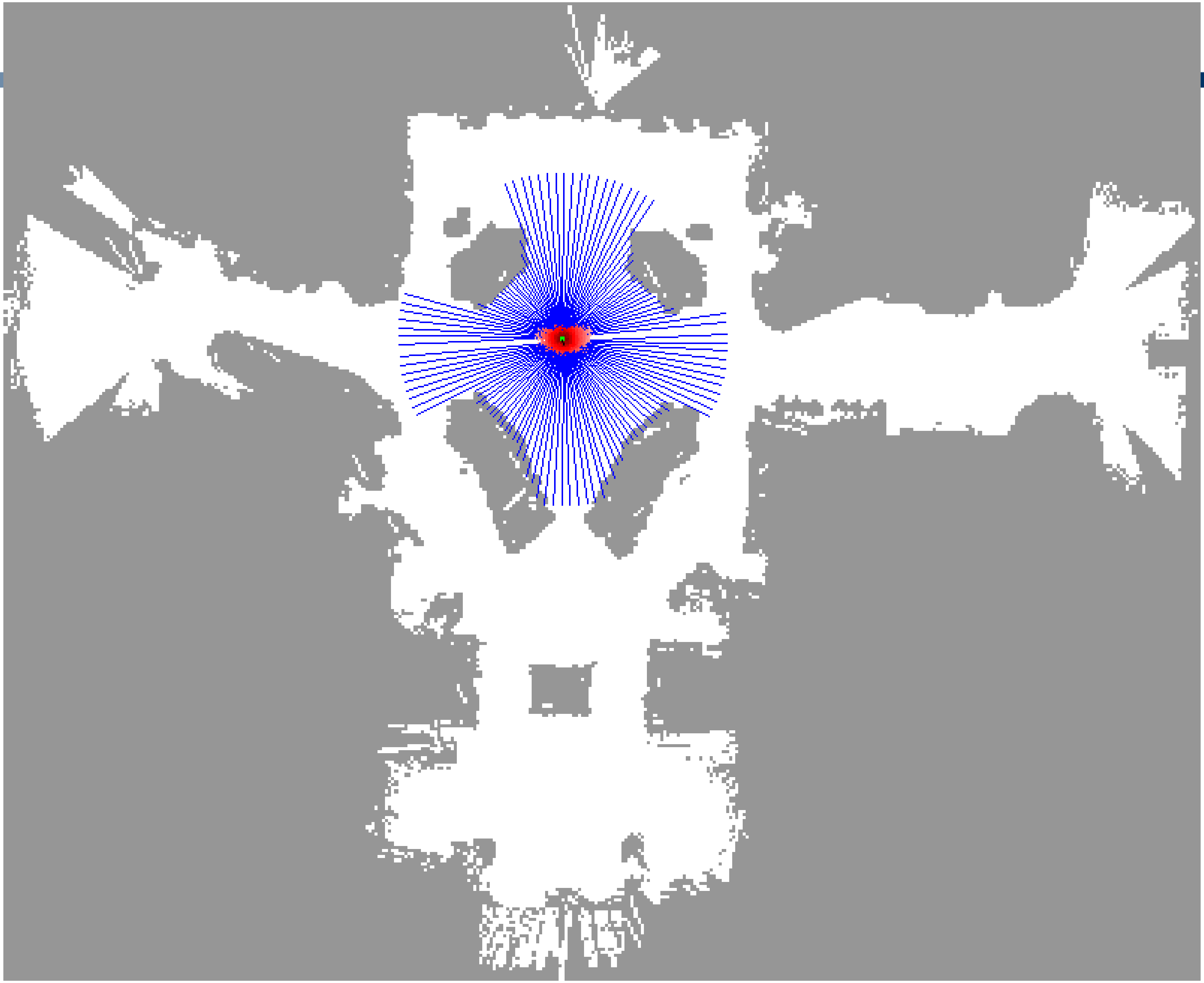




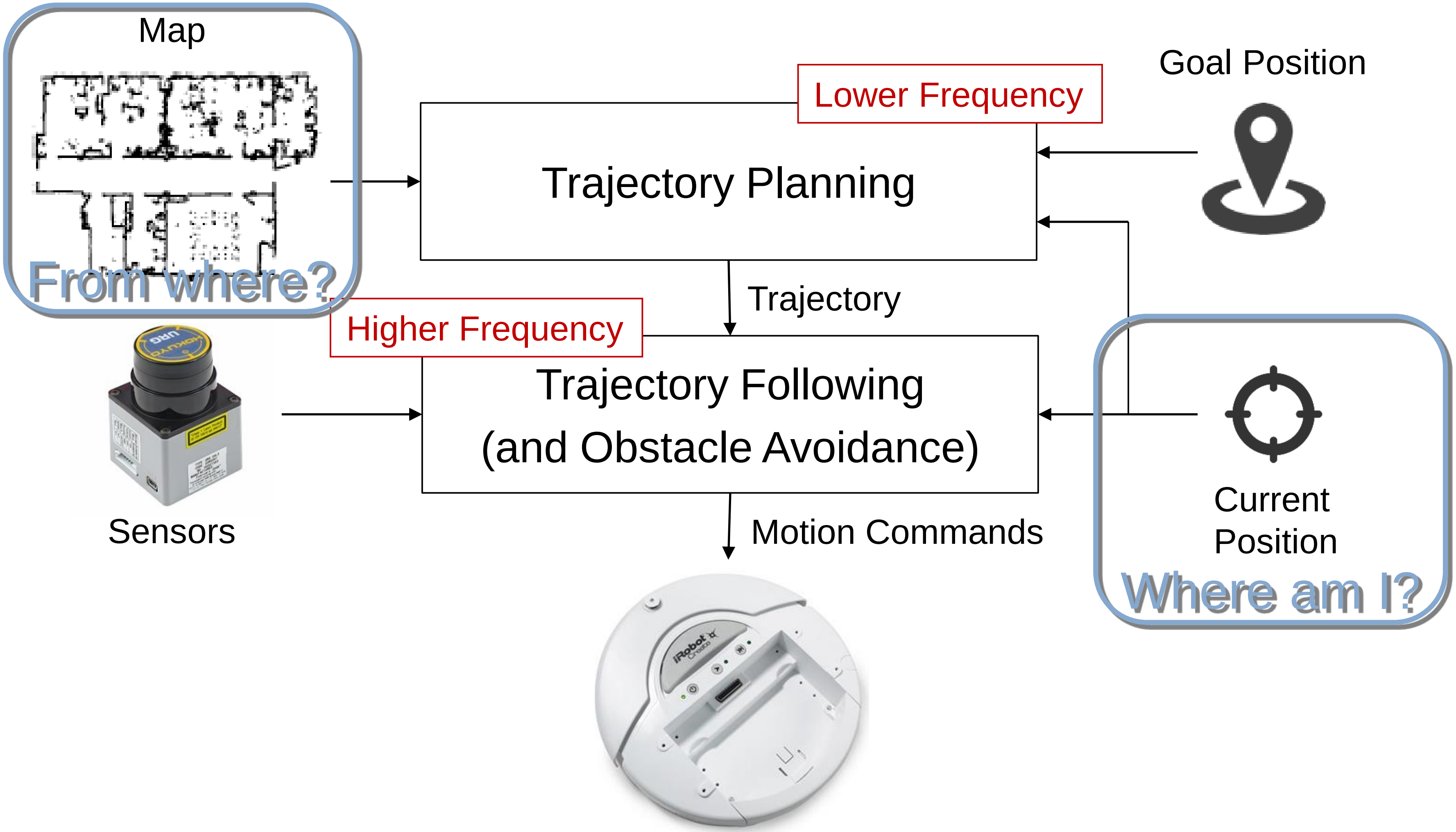






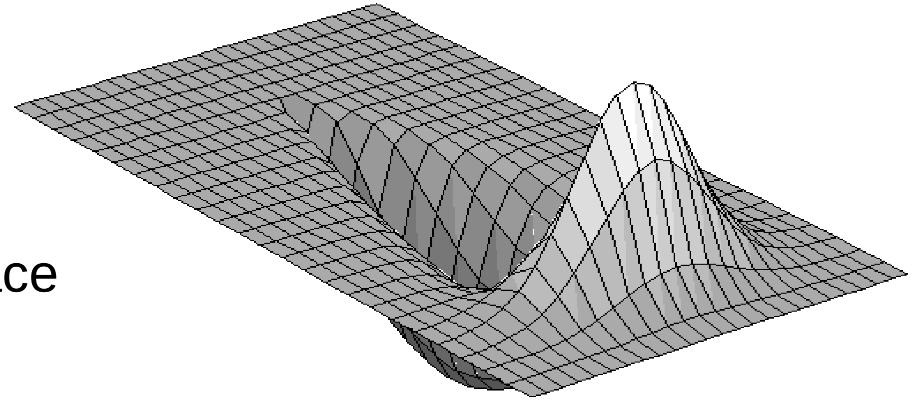


A Two Layered Approach



Matteo Matteucci – matteo.matteucci@polimi.it

Occupancy from Sonar Return



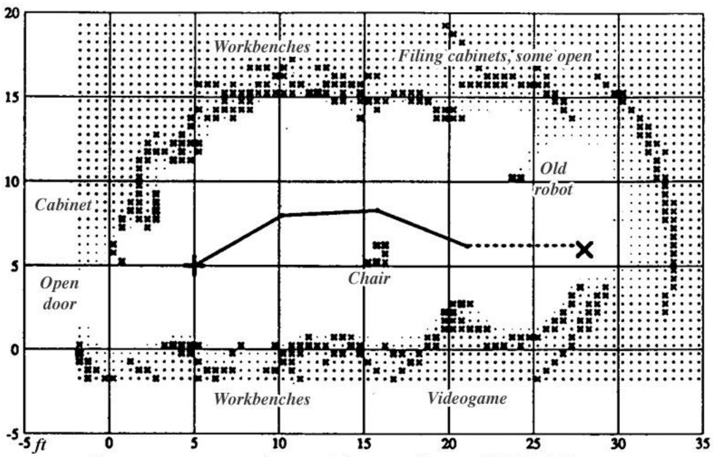
The most simple occupancy model uses

- A 2D Gaussian for information about occupancy
- Another 2D Gaussian for free space

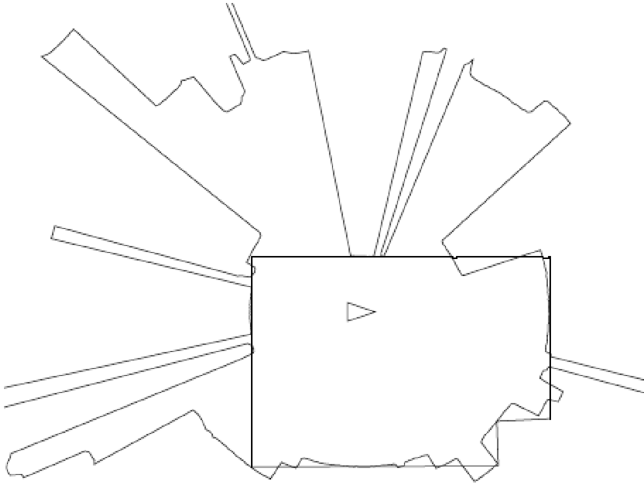
Sonar sensors present several issues

- A wide sonar cone creates noisy maps
- Specular (multi-path) reflections generates unrealistic measurements

Moravec 1984

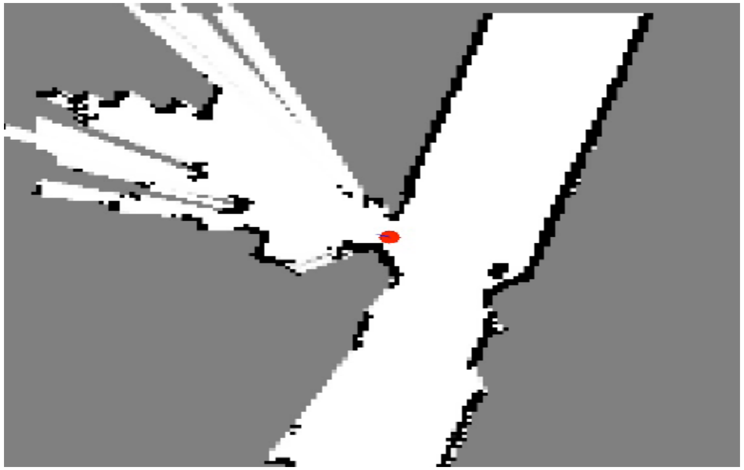


Room traverse by grid map from SONAR



2D Occupancy Grids

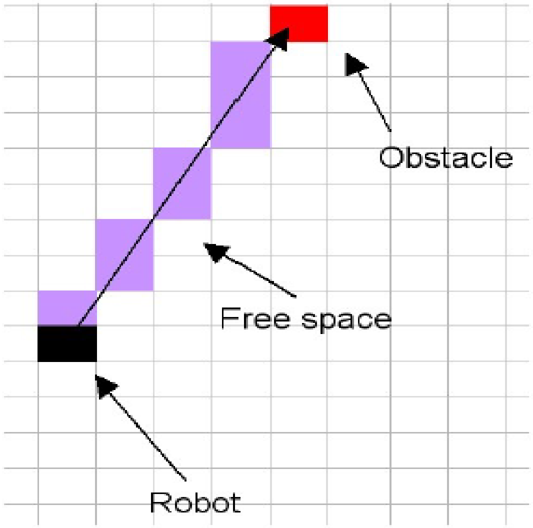
A simple 2D representation for maps



- Each cell is assumed independent
- Probability of a cell of being occupied estimated using Bayes theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\sim A)P(\sim A)}$$

Maps the environment as an array of cells



- Usual cell size 5 to 50cm
- Each cells holds the probability of the cell to be occupied
- Useful to combine different sensor scans and different sensor modalities

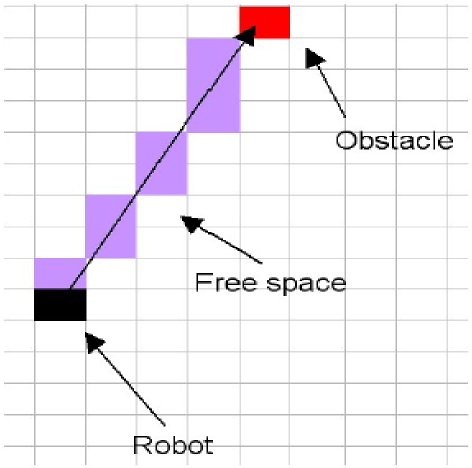
Occupancy Grid Cell Update

Let $occ(i,j)$ mean cell C_{ij} is occupied, then we have

- Probability: $p(occ(i,j))$ has range $[0,1]$
- Odds: $o(occ(i,j))$ has range $[0,+\infty)$

$$o(A) = \frac{P(A)}{P(\neg A)}$$

- Log odds: $\log o(occ(i,j))$ has range $(-\infty,+\infty)$
  - Each cell Cij holds the value $\log o\big(occ(i,j)\big)$
  - $Cij = 0$ corresponds to $p\big(occ(i,j)\big) = 0.5$



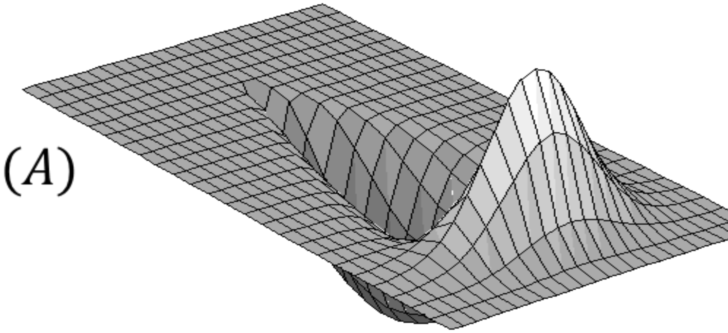
We will apply Bayes Law

- where *A* is *occ*(*i*,*j*)
- and *B* is an observation *r* = *D*

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

We can simplify this by using the log odds representation …

Occupancy by Log Odds



Lets consider Bayes law

- $o(A|B) = \frac{p(A|B)}{P(\neg A|B)} = \frac{p(B|A)P(A)}{P(B|\neg A)P(\neg A)} = \tau(B|A)o(A)$
- $\log o(A|B) = \log \tau(B|A) + \log o(A)$

To update the log odds of a cell at distance D

- $\log o(occ(i,j) \mid r = D) = \log \tau(\mathrm{r = D \mid occ(i,j)}) + \log o\left(occ(i,j)\right)$

Assume cell C_{ij} holds $\log o\left(occ(i,j)\right)$

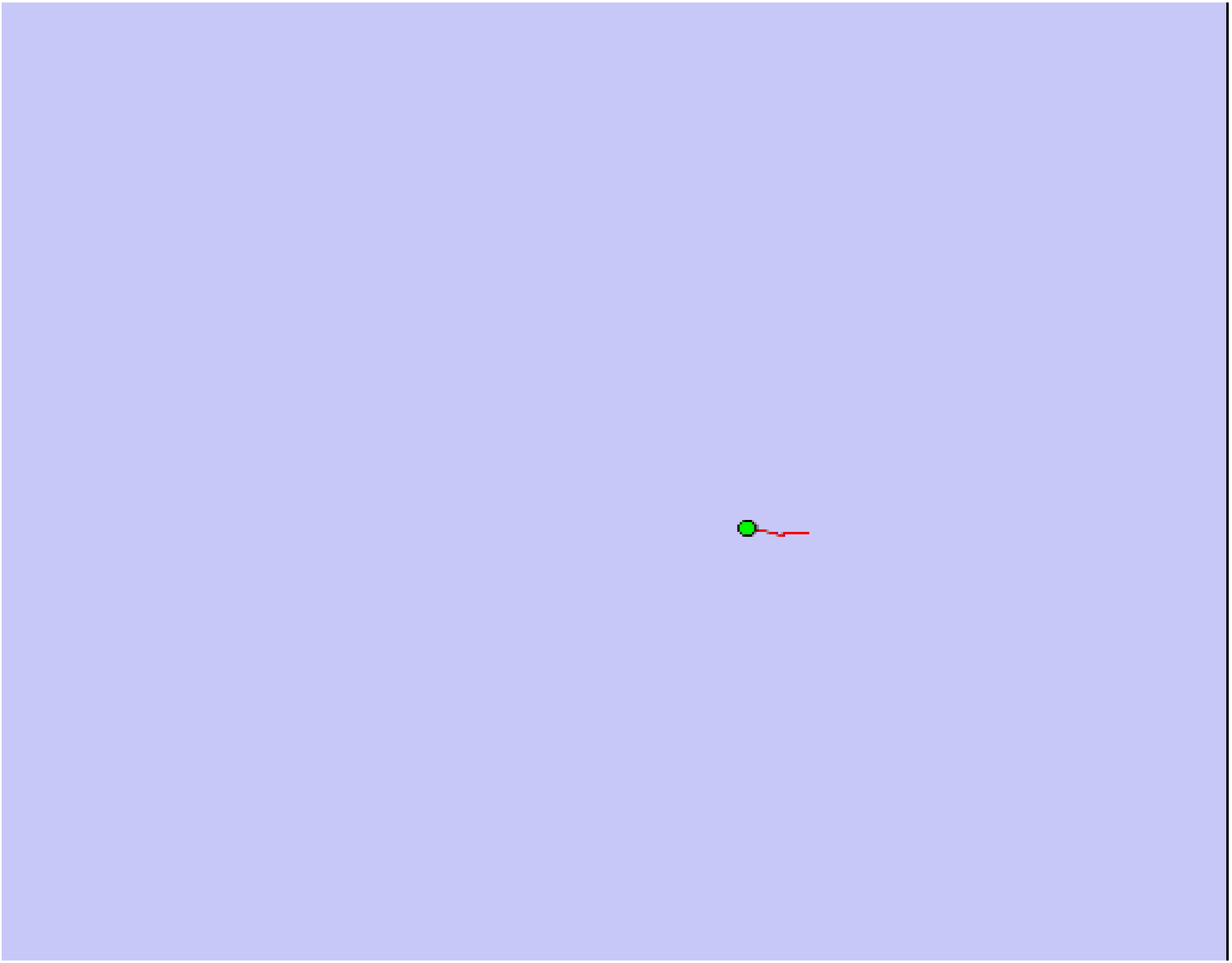
- Let be r the measurement from the sensor
- Let D be the distance of the cell
- For each cell C_{ij} accumulate evidence from each sensor reading

$$\tau(r = D|occ(i,j)) = \frac{p(r = D|occ(i,j))}{p(r = D|\neg occ(i,j))} \approx \frac{.06}{.005} = 12 \quad \rightarrow \quad log_2 \tau = 3.5$$

$$\tau(r > D|occ(i,j)) = \frac{p(r > D|occ(i,j))}{p(r > D|\neg occ(i,j))} \approx \frac{.45}{.90} = .5 \quad \rightarrow \quad log_2 \tau = -1$$

Matteo Matteucci – matteo.matteucci@polimi.it

Mapping with Raw Odometry (with known poses)



Maximize the likelihood of the *i-th* pose and map relative to the *(i-1)-th* pose and map.

$$\hat{x}_t = \arg\max_{x_t} \left\{ p(z_t \mid x_t, \hat{m}^{[t-1]}) \cdot p(x_t \mid u_{t-1}, \hat{x}_{t-1}) \right\}$$

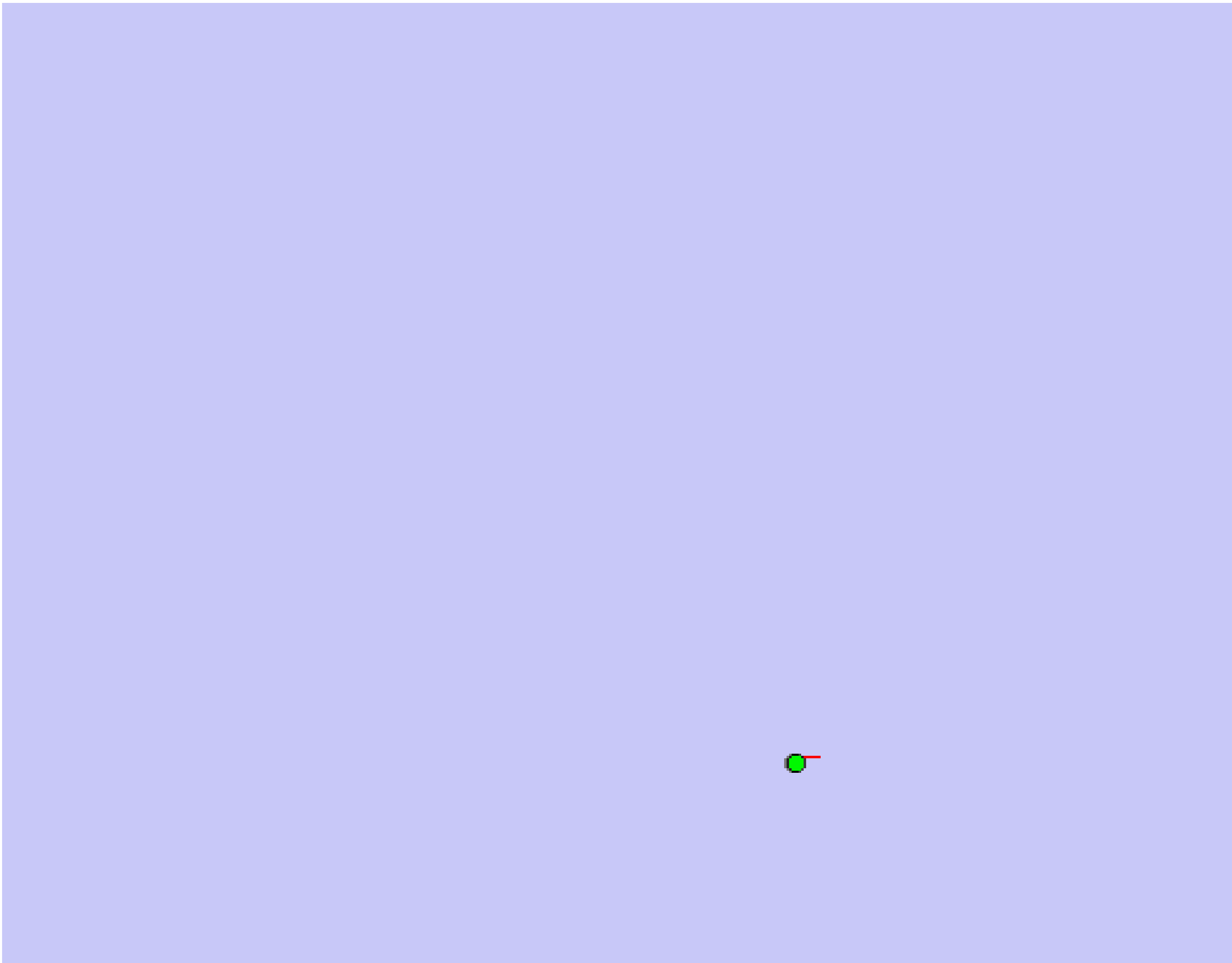
current measurement

robot motion

map constructed so far

Calculate the map $\hat{m}^{[t]}$ according to "mapping with known poses" based on the poses and observations.

Scan Matching Example



Full SLAM:

$$p(x_{1:t}, m \mid z_{1:t}, u_{1:t})$$

Estimates entire path and map!

Online SLAM:

$$p(x_t, m \mid z_{1:t}, u_{1:t}) = \int \int \dots \int p(x_{1:t}, m \mid z_{1:t}, u_{1:t})\, dx_1 dx_2 \dots dx_{t-1}$$

Estimates most recent pose and map!

Integrations typically done one at a time

SLAM: Simultaneous Localization and Mapping

Full SLAM:

$$p(x \quad m | z \quad u \quad)$$

Two famous example of this!

Extended Kalman Filter (EKF) SLAM

- Solves online SLAM problem
- Uses a linearized Gaussian probability distribution model

FastSLAM

- Solves full SLAM problem
- Uses a sampled particle filter distribution model

Online Sl

$$dx_{t-1}$$

Integrations typically done one at a time

Map with N landmarks:(3+2N)-dimensional Gaussian

$$Bel(x_t, m_t) = \left\langle \begin{pmatrix} x \\ y \\ \theta \\ l_1 \\ l_2 \\ \vdots \\ l_N \end{pmatrix}, \begin{pmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{x\theta} & \sigma_{xl_1} & \sigma_{xl_2} & \cdots & \sigma_{xl_N} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{y\theta} & \sigma_{yl_1} & \sigma_{yl_2} & \cdots & \sigma_{yl_N} \\ \sigma_{x\theta} & \sigma_{y\theta} & \sigma_\theta^2 & \sigma_{\theta l_1} & \sigma_{\theta l_2} & \cdots & \sigma_{\theta l_N} \\ \sigma_{xl_1} & \sigma_{yl_1} & \sigma_{\theta l_1} & \sigma_{l_1}^2 & \sigma_{l_1 l_2} & \cdots & \sigma_{l_1 l_N} \\ \sigma_{xl_2} & \sigma_{yl_2} & \sigma_{\theta l_2} & \sigma_{l_1 l_2} & \sigma_{l_2}^2 & \cdots & \sigma_{l_2 l_N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{xl_N} & \sigma_{yl_N} & \sigma_{\theta l_N} & \sigma_{l_1 l_N} & \sigma_{l_2 l_N} & \cdots & \sigma_{l_N}^2 \end{pmatrix} \right\rangle$$

$$\boxed{x_t = A_t x_{t-1} + B_t u_t + \varepsilon_t}$$

$$\boxed{p(x_t \mid u_t, x_{t-1}) = N\left(x_t; A_t x_{t-1} + B_t u_t, R_t\right)}$$

(E)KF-SLAM

Map with N landmarks:(3+2N)-dimensional Gaussian

$$Bel(x_t, m_t) = \left\langle \begin{pmatrix} x \\ y \\ \theta \\ l_1 \\ l_2 \\ \vdots \\ l_N \end{pmatrix}, \begin{pmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{x\theta} & \sigma_{xl_1} & \sigma_{xl_2} & \cdots & \sigma_{xl_N} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{y\theta} & \sigma_{yl_1} & \sigma_{yl_2} & \cdots & \sigma_{yl_N} \\ \sigma_{x\theta} & \sigma_{y\theta} & \sigma_\theta^2 & \sigma_{\theta l_1} & \sigma_{\theta l_2} & \cdots & \sigma_{\theta l_N} \\ \sigma_{xl_1} & \sigma_{yl_1} & \sigma_{\theta l_1} & \sigma_{l_1}^2 & \sigma_{l_1 l_2} & \cdots & \sigma_{l_1 l_N} \\ \sigma_{xl_2} & \sigma_{yl_2} & \sigma_{\theta l_2} & \sigma_{l_1 l_2} & \sigma_{l_2}^2 & \cdots & \sigma_{l_2 l_N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{xl_N} & \sigma_{yl_N} & \sigma_{\theta l_N} & \sigma_{l_1 l_N} & \sigma_{l_2 l_N} & \cdots & \sigma_{l_N}^2 \end{pmatrix} \right\rangle$$

$$\boxed{z_t = C_t x_t + \delta_t}$$

$$\boxed{p(z_t \mid x_t) = N\left(z_t; C_t x_t, Q_t\right)}$$

Bayes Filter: The Algorithm

$$Bel(x_t) = \eta \ P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \ Bel(x_{t-1}) \ dx_{t-1}$$

Algorithm Bayes_filter(*Bel(x), d*):

η=0

If *d* is a perceptual data item *z* then

For all *x* do

$$Bel'(x) = P(z \mid x) Bel(x)$$
$$\eta = \eta + Bel'(x)$$

For all *x* do

$$Bel'(x) = \eta^{-1} Bel'(x)$$

correction

Else if *d* is an action data item *u* then

prediction

For all *x* do

$$Bel'(x) = \int P(x \mid u, x') \ Bel(x') \ dx'$$

Return *Bel'(x)*

Kalman Filter Algorithm

Algorithm Kalman_filter(μ_{t-1}, Σ_{t-1}, u_t, z_t):

Prediction:

$$\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$$

$$\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$$

Correction:

$$K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$

$$\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$$

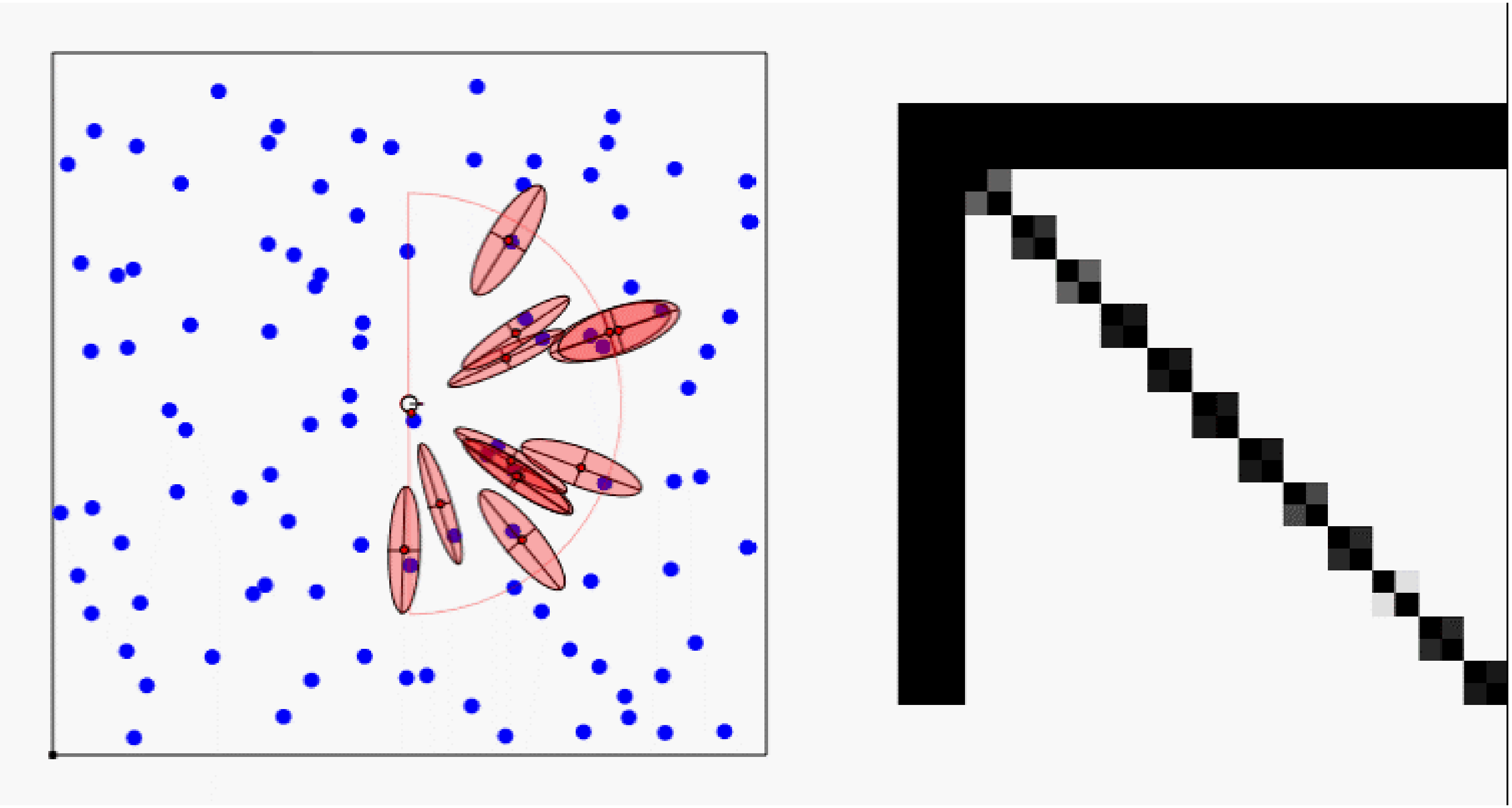
$$\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$$

Return μ_t, Σ_t

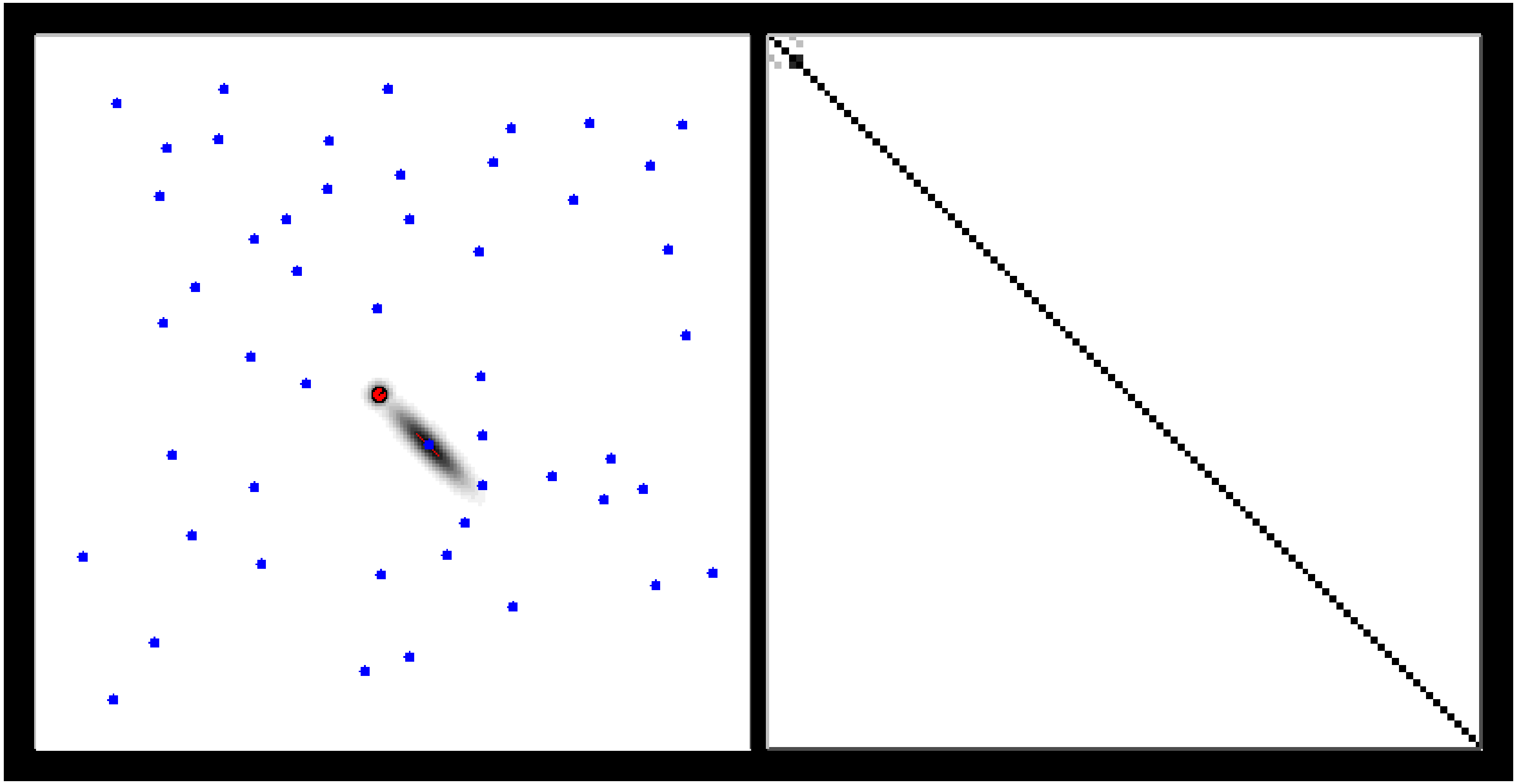
$$Bel(x_t, m_t) = \left\langle \begin{pmatrix} x \\ y \\ \theta \\ l_1 \\ l_2 \\ \vdots \\ l_N \end{pmatrix}, \begin{pmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{x\theta} & \sigma_{xl_1} & \sigma_{xl_2} & \cdots & \sigma_{xl_N} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{y\theta} & \sigma_{yl_1} & \sigma_{yl_2} & \cdots & \sigma_{yl_N} \\ \sigma_{x\theta} & \sigma_{y\theta} & \sigma_\theta^2 & \sigma_{\theta l_1} & \sigma_{\theta l_2} & \cdots & \sigma_{\theta l_N} \\ \sigma_{xl_1} & \sigma_{yl_1} & \sigma_{\theta l_1} & \sigma_{l_1}^2 & \sigma_{l_1 l_2} & \cdots & \sigma_{l_1 l_N} \\ \sigma_{xl_2} & \sigma_{yl_2} & \sigma_{\theta l_2} & \sigma_{l_1 l_2} & \sigma_{l_2}^2 & \cdots & \sigma_{l_2 l_N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{xl_N} & \sigma_{yl_N} & \sigma_{\theta l_N} & \sigma_{l_1 l_N} & \sigma_{l_2 l_N} & \cdots & \sigma_{l_N}^2 \end{pmatrix} \right\rangle$$

Classical Solution – The EKF

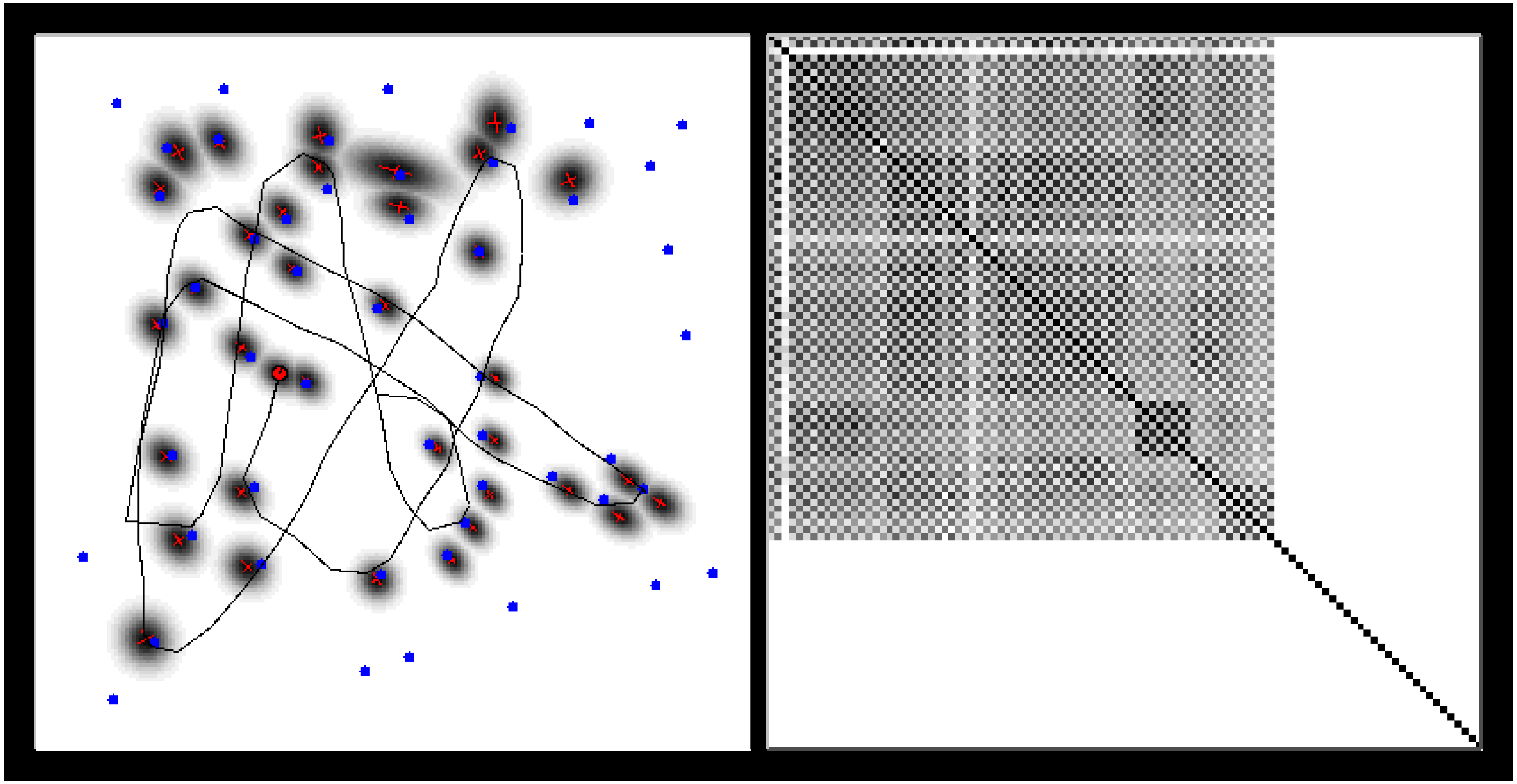
Approximate the SLAM posterior with a high-dimensional Gaussian



Blue path = true path **Red path** = estimated path **Black path** = odometry

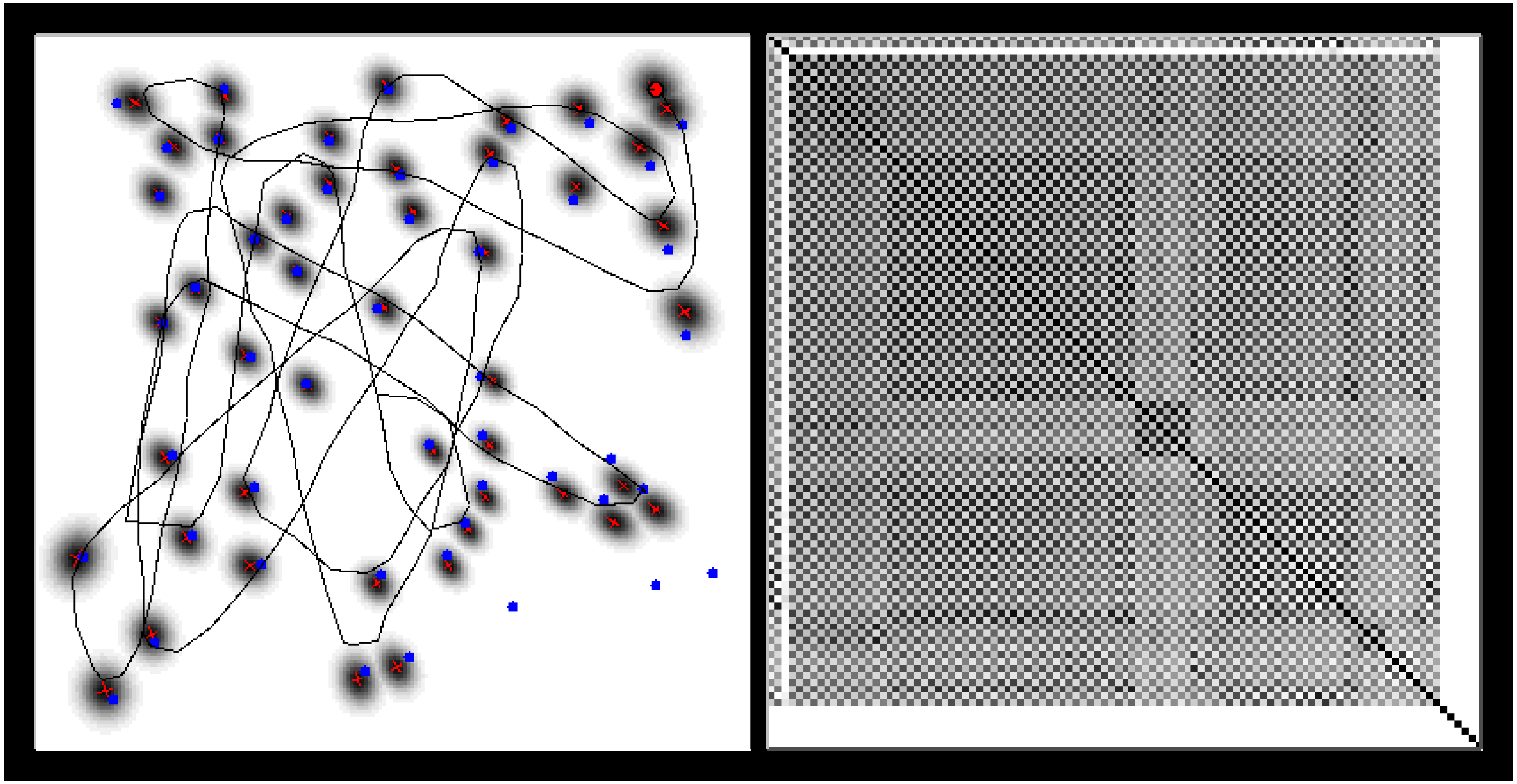


Map Correlation matrix



Map

Correlation matrix



Map Correlation matrix

Properties of KF-SLAM (Linear Case)

Theorem: [Dissanayake et al., 2001]

The determinant of any sub-matrix of the map covariance matrix decreases monotonically as successive observations are made.

Theorem:

In the limit the landmark estimates become fully correlated

Are we happy about this?

- Quadratic in the number of landmarks: $O(n^2)$
- Convergence results for the linear case.
- Can diverge if nonlinearities are large!
- Have been applied successfully in large-scale environments.
- Approximations reduce the computational complexity.

EKF Drawbacks

EKF-SLAM works pretty well but ...

- EKF-SLAM employs linearized models of nonlinear motion and observation models and so inherits many caveats.
- Computational effort is demand because computation grows quadratically with the number of landmarks.

Possible solutions

- Local submaps [Leonard & al 99, Bosse & al 02, Newman & al 03]
- Sparse links (correlations) [Lu & Milios 97, Guivant & Nebot 01]
- Sparse extended information filters [Frese et al. 01, Thrun et al. 02]
- Thin junction tree filters [Paskin 03]
- Rao-Blackwellisation (FastSLAM) [Murphy 99, Montemerlo et al. 02, Eliazar et al. 03, Haehnel et al. 03]
  - Represents nonlinear process and non-Gaussian uncertainty
  - Rao-Blackwellized method reduces computation

The FastSLAM Idea (Full SLAM)

In the general case we have

$$p(x_t, m \mid z_t) \neq P(x_t \mid z_t) P(m \mid z_t)$$

However if we consider the full trajectory X_t rather than the single pose $x_{t.}$

$$p(X_t, m \mid z_t) = P(X_t \mid z_t) P(m \mid X_t, z_t)$$

In FastSLAM, the trajectory X_t is represented by particles X_t(i) while the map is represented by a factorization called Rao-Blackwellized Filter

$$P(m \mid X_t^{(i)}, z_t) = \prod_j^M P(m_j \mid X_t^{(i)}, z_t)$$

- $P(X_t \mid z_t)$ through particles
- $P(m \mid X_t, z_t)$ using an EKF

FastSLAM Formulation

Decouple map of features from pose ...

- Each particle represents a robot trajectory
- Feature measurements are correlated thought the robot trajectory
- If the robot trajectory is known all of the features would be uncorrelated
- Treat each pose particle as if it is the true trajectory, processing all of the feature measurements independently

poses map observations & movements

$$p(x_{1:t}, l_{1:m} \mid z_{1:t}, u_{0:t-1}) =$$

$$\underbrace{p(x_{1:t} \mid z_{1:t}, u_{0:t-1})}_{\text{Robot path posterior}} \cdot \underbrace{p(l_{1:m} \mid x_{1:t}, z_{1:t})}_{\text{Landmark positions}}$$

SLAM posterior

Factored Posterior: Rao-Blackwellization

$$p(x_{1:t}, l_{1:m} \mid z_{1:t}, u_{0:t-1})$$
$$= p(x_{1:t} \mid z_{1:t}, u_{0:t-1}) \cdot p(l_{1:m} \mid x_{1:t}, z_{1:t})$$
$$= p(x_{1:t} \mid z_{1:t}, u_{0:t-1}) \cdot \prod_{i=1}^{M} p(l_i \mid x_{1:t}, z_{1:t})$$

Robot path posterior (localization problem)

Conditionally independent landmark positions

Dimension of state space is drastically reduced by factorization making particle filtering possible

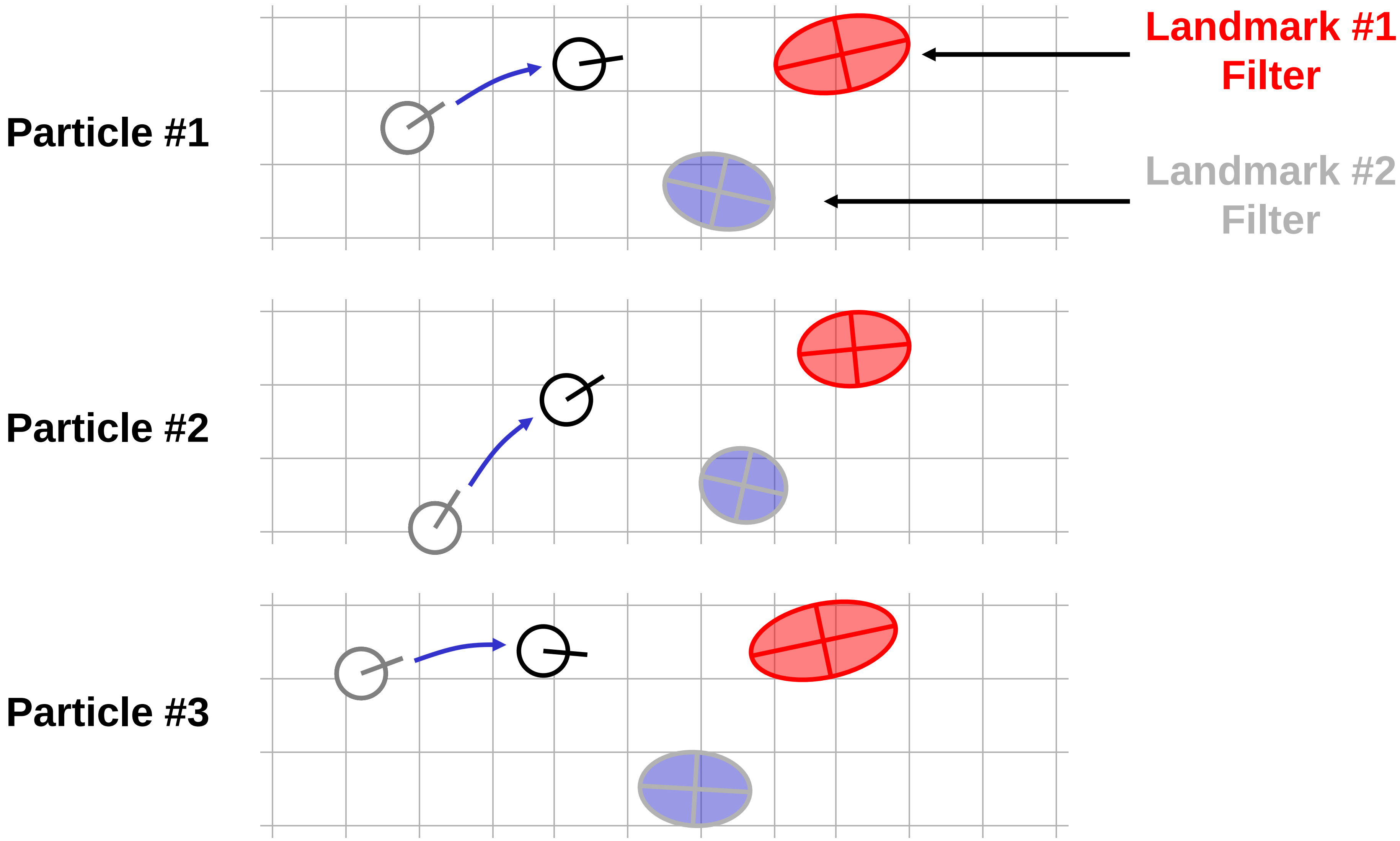
$$p(x_{1:t}, l_{1:m} \mid z_{1:t}, u_{0:t-1}) =$$
$$p(x_{1:t} \mid z_{1:t}, u_{0:t-1}) \cdot \prod_{i=1}^{M} p(l_i \mid x_{1:t}, z_{1:t})$$

FastSLAM in Practice

- Rao-Blackwellized particle filtering based on landmarks [Montemerlo et al., 2002]
- Each landmark is represented by a 2x2 Extended Kalman Filter (EKF)
- Each particle therefore has to maintain M EKFs

Particle #1	x, y, θ	**Landmark 1**	**Landmark 2**	**…**	**Landmark M**
Particle #2	x, y, θ	**Landmark 1**	**Landmark 2**	**…**	**Landmark M**
⋮					
Particle N	x, y, θ	**Landmark 1**	**Landmark 2**	**…**	**Landmark M**

FastSLAM – Action Update

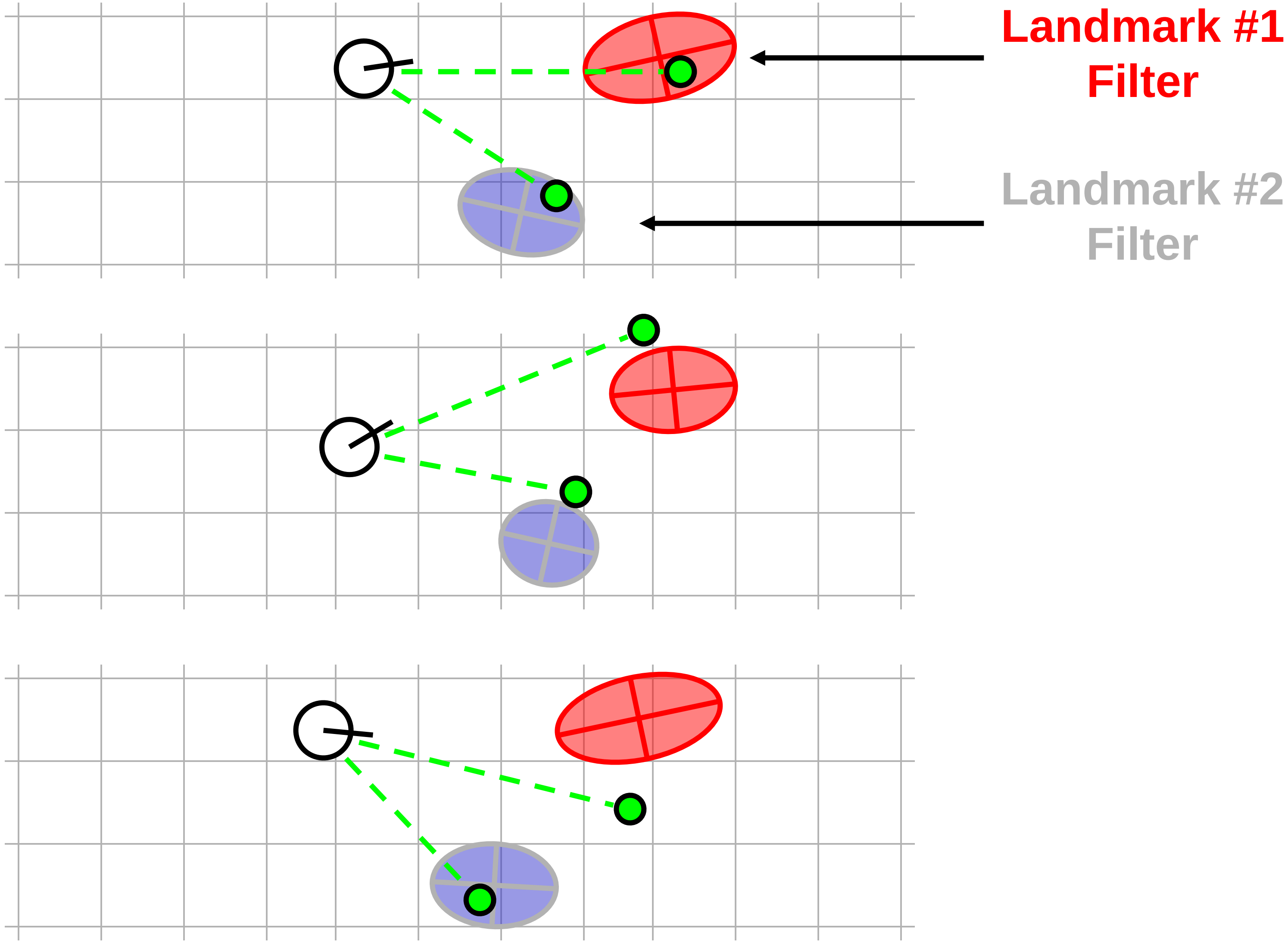


FastSLAM – Sensor Update

Particle #1

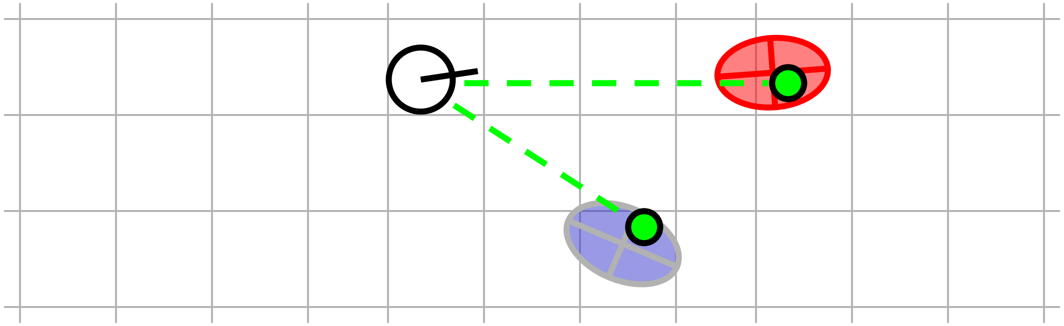
Particle #2

Particle #3



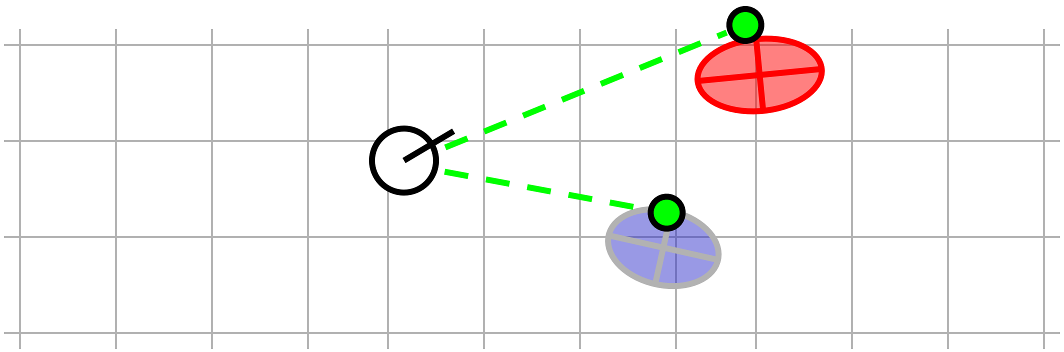
FastSLAM – Sensor Update

Particle #1



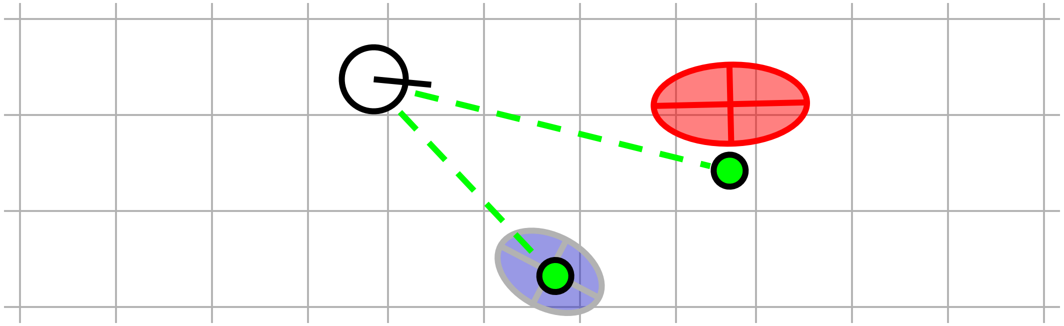
Weight = 0.8

Particle #2



Weight = 0.4

Particle #3



Weight = 0.1

FastSLAM Complexity

Update robot particles based on control u_{t-1}

$O(N)$

Constant time per particle

Incorporate observation z_t into Kalman filters

$O(N \cdot \log(M))$

Log time per particle

Resample particle set

$O(N \cdot \log(M))$

Log time per particle

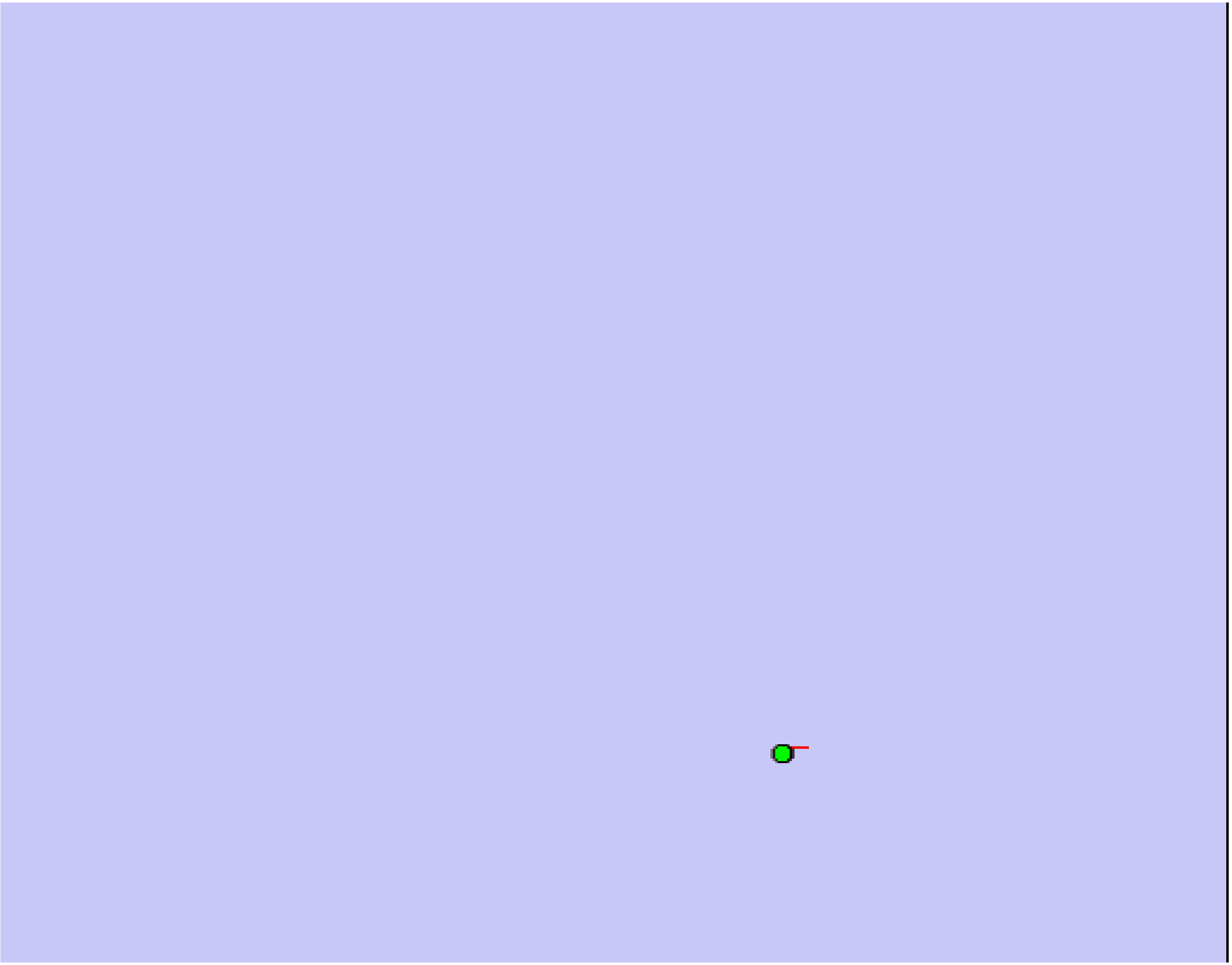
$O(N \cdot \log(M))$

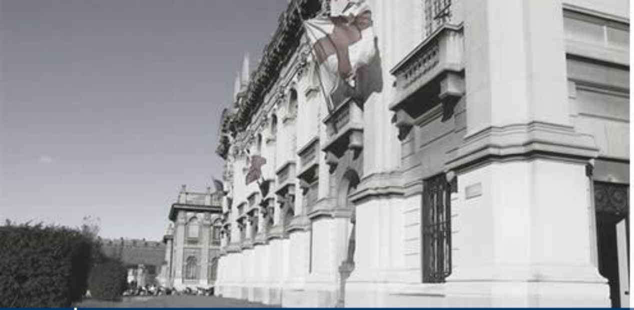
Log time per particle

N = Number of particles

M = Number of map features

Fast-SLAM Example





POLITECNICO DI MILANO





Cognitive Robotics –SLAM with Lasers

Matteo Matteucci – matteo.matteucci@polimi.it